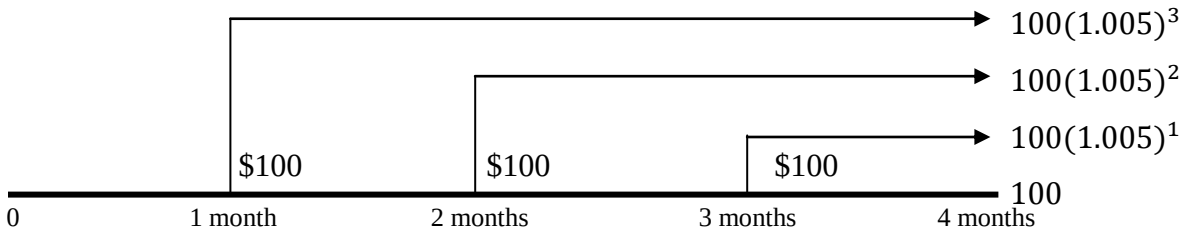
MCR 3U

PART 1: ORDINARY ANNUITIES

An ***annuity*** is an investment in which regular payments (payments of equal value made at equal time periods) are deposited into an account. An ***ordinary annuity*** is one in which payments are made at the end of every payment period and interest is compounded at the end of the same payment period.

Consider $100 invested at the end of each month for 4 months, earning interest at 6%/a compounded monthly.



The sum of all terms at the end of the above lines would represent the total of the annuity after a 4-month period. The above calculation is not too tedious, whereas, if the investment is for 1 year, the sum of 12 individual calculations would be performed.

The following formula accounts for the total of all terms in an investment where regular payments are being made to an account. The formula is used to determine the accumulated amount of an ordinary annuity.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

Where
- R = regular payment per period
- A = accumulated amount after n periods
- i = interest rate per period
- n = number of periods

Rate Per Period

As with any financial formula that involves a rate, it is important to make sure that the rate is consistent with the other variables in the formula. If the payment is per month, then the rate needs to be per month, and similarly, the rate would need to be the annual rate if the payment is annual.

An example would be an annuity that has a 12% annual rate and payments are made monthly. The monthly rate of 1% would need to be used in the formula.

EXAMPLE: A $500 payment is made every month in an account for 5 years at an interest rate of 6% per annum compounded monthly. How much money is in the account at the end of the 5 years and how much interest is earned?

EXAMPLE: You wish to accumulate $500 000 in an account by the end of 10 years. If a bank is offering an interest rate of 9%/a compounded semimonthly, what regular payment should you make to meet your goal?

PART 2: PRESENT VALUE OF AN ANNUITY

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

Where PV = present value of a loan/ bursury/ lottery
R = regular payment per period
i = interest rate per period
n = number of periods

The PRESENT VALUEOF ANNUITY formula determines the value of a series of future periodic payments at a given time. The present value of annuity formula relies on the concept of time value of money, in that one dollar present day is worth more than that same dollar at a future date.

EXAMPLE: A car is purchased for $50 000 today by making a downpayment of 10%, then financing (taking a loan) the rest of the purchase over a 5-year period. The loan must be paid in monthly payments at an interest rate of 3% per annum compounded monthly. How much is each monthly payment?

EXAMPLE: You win a lottery in which you must choose 1 of 2 options:

- Receive $1000 per week for 15 years
- Receive $100 000 now

Which option should you choose, given that a bank offers an interest rate of 4%/a compounded weekly?