MCR 3U

PART 1: ARITHMETIC SEQUENCES

Arithmetic sequences: sequences in which successive terms are determined by adding or subtracting the same value.

Examples:

1. 6, 10, 14, 18, ...
2. 7, -2, -11, -20, ...
3. $2, \frac{4}{3}, \frac{2}{3}, 0, \frac{-2}{3}, \ldots$
4. $x-2,\ 2x+1,\ 3x+4, \ldots$

In any sequence,

$t_1 = first\ term$

$t_2 = second\ term$

...

$t_n = nth\ term\ or\ general\ term\ or\ general\ formula$

EXAMPLE 1: Given $t_n = 3 - 8n$, find the first 4 terms.

EXAMPLE 2: Given $t_n = \frac{4n+3}{2}$, find the fifth and twentieth terms.

In general, if the common difference between terms is represented by the symbol "d", the general formula for an arithmetic sequence is

$$\boldsymbol{t_n = t_1 + (n-1)(d)}$$

where n = number of terms, or the position of a term

d = common difference

PROOF:

The general arithmetic sequence is...

EXAMPLE 3: Given the sequence of terms 4, -2, -8, ... find the general formula. Find t_{50}.

EXAMPLE 4: How many terms are in the sequence 12, 20, 28, ..., 316?

EXAMPLE 5: In an arithmetic sequence, the seventh term is 121 and the fifteen term is 193. Find the first term and the common difference. Then determine the general formula for the sequence.

EXAMPLE 6: Brandon earns $20 000 in his first year as a waiter. His annual salary increases each year by $4 000.

A) Write a general formula for Brandon's salary in any given year.

B) Determine Brandon's salary in his twelfth year of service.

PART 2: ARITHMETIC SERIES

1, 3, 5, 7, ... is called an Arithmetic Sequence, since it has a common difference of $3 - 1 = 2$.

An ***ARITHMETIC SERIES*** is the sum of the terms of an arithmetic sequence. In the sequence above, $1 + 3 + 5 + 7 + \ldots$ would represent the arithmetic series.

The symbol $\boldsymbol{S_n}$ represents the **SUM of the first $\boldsymbol{n}$ terms** of a series.
For instance, S_5 represents the sum of the first 5 terms.
For the above series, $S_5 = 1 + 3 + 5 + 7 + 9 = 25$.

In some cases, such as the one above, the calculation is simple; however, calculating S_{32} using the same method as above is inefficient.

Below, we find the proof of the formula for S_n of an arithmetic series.

First, consider the case of $\boldsymbol{S_5}$ for the series $1 + 3 + 5 + 7 + 9$:

$$S_5 = 1 + 3 + 5 + 7 + 9$$
$$S_5 = 9 + 7 + 5 + 3 + 1$$

Adding the 2 equations together produces... $2S_5 = 10 + 10 + 10 + 10 + 10$

Solving for S_5... $2S_5 = 50$

$$S_5 = 25$$

Using general symbols, the formula for $\boldsymbol{S_n}$ of an arithmetic series developed.

① $S_n = t_1 + (t_1 + d) + (t_1 + 2d) + (t_1 + 3d) + \cdots + (t_1 - d) + t_n$

② $S_n = t_n + (t_n - d) + (t_n - 2d) + (t_n - 3d) + \cdots + (t_1 + d) + t_1$

①+② $2S_n = (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \cdots + (t_1 + t_n) + (t_1 + t_n)$

Simplify: $2S_n = n(t_1 + t_n)$

$$\boldsymbol{S_n = \frac{n(t_1 + t_n)}{2}}$$ where $n = \textit{number of terms in the sum}$

t_1 *and* t_n *are the first and last terms*

EXAMPLE 1: Find S_{24} for the series defined by $t_n = 4 - 3n$.

EXAMPLE 2: Find the sum of the series $6 + 10 + 14 + \cdots + 170$.

EXAMPLE 3: Find the sum of the first 50 terms of the series $-7 - 5 - 3 - 1 - \ldots$

EXAMPLE 4: Given an arithmetic series has $S_5 = 35$ and $S_{12} = 252$, determine the first term and the common difference. Also, determine the general formula for the sequence.