

MDM 4U

PROBLEM SOLVING with COMBINATIONS

RECALL the following formulas:

① $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

- n distinct objects available, choosing r of the objects where order is not important.

② $2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n-1} + \binom{n}{n}$

- total number of subsets of a set containing n elements.
- total number of possible selections from a set of n distinct elements

③ $2^n - 1$

- total number of subsets of a set containing n elements, excluding the null set.
- total number of possible selections from a set of n distinct elements; at least 1 selection must be made.

④ $(n+1)(m+1)(p+1)\dots - 1$

- from a set of n objects that are alike, m objects that are alike, p objects that are alike, in which a selection of at least 1 item is made.

TO COMPARE THE USE OF FORMULAS ③ AND ④:

Consider the set $A = \{a, b, c, d\}$. The subsets of set A , excluding $\{\}$ are:

$\{a\}, \{b\}, \{c\}, \{d\}$

$\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

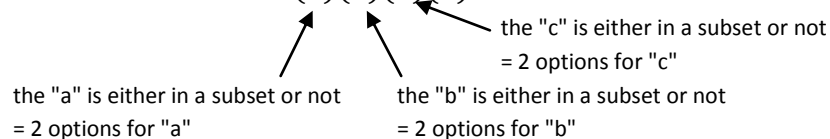
$\{a, b, c, d\}$

There are $2^4 - 1 = 15$ subsets.

In set A , all elements are distinct, so all 1-element subsets, all 2-element subsets, and all 3-element subsets are different. There are no subsets that are alike.

Each element of set A is either in a subset or not; the "a" is either in a subset or not, the "b" is either in a subset or not, and so on.

Another way to write the formula $2^4 - 1$ is $(2)(2)(2)(2) - 1 = 15$ subsets.



Consider the set $B = \{a, a, a, d\}$. The subsets of set B, excluding $\{\}$ are:

$\{a\}$, $\{a\}$, $\{a\}$, $\{d\}$

$\{a, a\}$, $\{a, a\}$, $\{a, d\}$, $\{a, a\}$, $\{a, d\}$, $\{a, d\}$

$\{a, a, a\}$, $\{a, a, d\}$, $\{a, a, d\}$, $\{a, a, d\}$

$\{a, a, a, d\}$

In set B, some of the elements are alike, so some of the subsets will contain the same list.

If we cross out the duplicate subsets, we have only 7 distinct subsets of set B.

$\{a\}$, ~~$\{a\}$~~ , ~~$\{a\}$~~ , $\{d\}$

$\{a, a\}$, ~~$\{a, a\}$~~ , $\{a, d\}$, ~~$\{a, a\}$~~ , ~~$\{a, d\}$~~ , ~~$\{a, d\}$~~

$\{a, a, a\}$, $\{a, a, d\}$, ~~$\{a, a, d\}$~~ , ~~$\{a, a, d\}$~~

$\{a, a, a, d\}$

In order to determine the appropriate number of distinct subsets of set B in which some items are alike, we approach the problem in the same manner as set A, in which there are distinct items...

Subsets of B will contain either no "a"s, one "a", two "a"s, or all three "a"s; that is, there are 4 options of a's in subsets. In terms of the "d", either a subset contains the "d" or not; that is, there are 2 options.

In other words, the number of distinct subsets is...

$$(3 + 1)(1 + 1) - 1 = (4)(2) - 1 = 7 \text{ subsets}$$

EXAMPLES:

- ① How many selections of cookies are possible from a container with 6 chocolate chip, 4 raisin, and 3 peanut butter cookies?
- ② How many selections of candies are possible from a box of 16 different candies?
- ③ If you reach into your pocket for some change, how many different selections are possible if you have 3 nickels, 2 dimes, 6 quarters and 4 loonies?
- ④ A small bag consists of 15 peanuts, 8 walnuts, 6 cashews and 10 hazel nuts. How many selections can be made if... [caution: this question may contain nuts!]
 - A) there are no restrictions?
 - B) at least 1 of the nuts is a peanut?
 - C) at least 1 of the nuts is a hazel nut?