MDM 4U
PROBLEM SOLVING with COMBINATIONS

RECALL the following formulas:

① $\binom{\boldsymbol{n}}{\boldsymbol{r}} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!\boldsymbol{r}!}$

- n distinct objects available, choosing r of the objects where order is not important.

② $\mathbf{2}^{\boldsymbol{n}}$ = $\binom{\boldsymbol{n}}{\mathbf{0}} + \binom{\boldsymbol{n}}{\mathbf{1}} + \binom{\boldsymbol{n}}{\mathbf{2}} + \binom{\boldsymbol{n}}{\mathbf{3}} + \cdots + \binom{\boldsymbol{n}}{\boldsymbol{n}-\mathbf{1}} + \binom{\boldsymbol{n}}{\boldsymbol{n}}$

- total number of subsets of a set containing n elements.
- total number of possible selections from a set of n distinct elements

③ $\mathbf{2}^{\boldsymbol{n}} - \mathbf{1}$

- total number of subsets of a set containing n elements, excluding the null set.
- total number of possible selections from a set of n distinct elements; at least 1 selection must be made.

④ $(\boldsymbol{n}+\mathbf{1})(\boldsymbol{m}+\mathbf{1})(\boldsymbol{p}+\mathbf{1})\ldots - \mathbf{1}$

- from a set of n objects that are alike, m objects that are alike, p objects that are alike, in which a selection of at least 1 item is made.

TO COMPARE THE USE OF FORMULAS ③ AND ④:

Consider the set $A = \{a, b, c, d\}$. The subsets of set A, excluding { } are:
{a}, {b}, {c}, {d}
{a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}
{a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}
{a, b, c, d}
There are $2^4 - 1 = 15$ subsets.

In set A, all elements are distinct, so all 1-element subsets, all 2-element subsets, and all 3-element subsets are different. There are no subsets that are alike.
Each element of set A is either in a subset or not; the "a" is either in a subset or not, the "b" is either in a subset or not, and so on.

Another way to write the formula $2^4 - 1$ is $(2)(2)(2)(2) - 1 = 15\ subsets.$

the "c" is either in a subset or not
= 2 options for "c"

the "a" is either in a subset or not
= 2 options for "a"

the "b" is either in a subset or not
= 2 options for "b"

Consider the set $B = \{a, a, a, d\}$. The subsets of set B, excluding { } are:
{a}, {a}, {a}, {d}
{a, a}, {a, a}, {a, d}, {a, a}, {a, d}, {a, d}
{a, a, a}, {a, a, d}, {a, a, d}, {a, a, d}
{a, a, a, d}

In set B, some of the elements are alike, so some of the subsets will contain the same list.
If we cross out the duplicate subsets, we have only 7 distinct subsets of set B.
{a}, ~~{a},~~ ~~{a},~~ {d}
{a, a}, ~~{a, a},~~ {a, d}, ~~{a, a}, {a, d}, {a, d}~~
{a, a, a}, {a, a, d}, ~~{a, a, d}, {a, a, d}~~
{a, a, a, d}

In order to determine the appropriate number of distinct subsets of set B in which some items are alike, we approach the problem in the same manner as set A, in which there are distinct items...
Subsets of B will contain either no "a"s, one "a", two "a"s, or all three "a"s; that is, there are 4 options of a's in subsets. In terms of the "d", either a subset contains the "d" or not; that is, there are 2 options.
In other words, the number of distinct subsets is...

$$(3+1)(1+1) - 1 = (4)(2) - 1 = \;\; 7\ subsets$$

EXAMPLES:

① How many selections of cookies are possible from a container with 6 chocolate chip, 4 raisin, and 3 peanut butter cookies?

② How many selections of candies are possible from a box of 16 different candies?

③ If you reach into your pocket for some change, how many different selections are possible if you have 3 nickels, 2 dimes, 6 quarters and 4 loonies?

④ A small bag consists of 15 peanuts, 8 walnuts, 6 cashews and 10 hazel nuts. How many selections can be made if... [caution: this question may contain nuts!]

A) there are no restrictions?
B) at least 1 of the nuts is a peanut?
C) at least 1 of the nuts is a hazel nut?