MDM 4U
COMBINATIONS

Given a set containing 5 items, how many subsets contain 3 elements?

Consider the set A = {a, b, c, d, e}, where n(A) = 5. List all subsets containing 3 elements.

{a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e} -- 10 subsets.

Note that the list of 3-element subsets are all distinct; that is, there are no 2 subsets which contain the same elements. The order of the elements found in any subset is not significant as the elements are a grouping of items without order.

If we apply the formula for a permutation (in which order DOES matter), we would calculate... $P(5,3) = \frac{5!}{2!} = 60$. Since we wish to determine the distinct subsets of 3 elements, we would "divide" out the factor which produces those sets that have the same subsets of elements but in different orders.

That is, since there are 10 subsets of 3 elements taken from 5 items,

$$10 = \frac{60}{6} = \frac{P(5,3)}{3!} = a\ combination\ of\ 3\ elements\ taken\ from\ 5\ items\ without\ regard\ to\ order$$

We define the ***COMBINATION*** of "n" items, taking "r" of them without regard to order by the formula:

$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

EXAMPLE 1:

A hand of poker contains 5 cards taken from a standard deck of 52 cards. How many hands...

A) are possible?
B) contain 3 queens and 2 kings?
C) contain 1 red card?
D) contain at least one face card?
E) contain all of the aces?

EXAMPLE 2:

Six men and six women are planning a trip using 2 vehicles, each with a capacity of 6 people. How many arrangements of the people...

A) are possible?
B) have Mike and Brenda together?
C) have Mike and Brenda separated?
D) have equal numbers of men and women in each vehicle?