MCV 4U
DERIVATIVES of COMPOSITE FUNCTIONS

- Used for composite functions in the form $F(x) = f\big(g(x)\big)$ or $F = f \circ g$.

$$\frac{dy}{dx} = \frac{dy}{da} \cdot \frac{da}{dx} \longrightarrow$$ Note the product between the derivative symbols.

EXAMPLE: Determine the derivative of $y = 4(x^2 + x - 2)^3$.

We can determine $\frac{dy}{dx}$ by method of substitution and applying the Chain Rule as shown above.

Let $a = x^2 + x - 2$, then by substitution, $y = 4a^3$.

If $y = 4a^3 \longrightarrow$ then $\frac{dy}{da} = 12a^2$.

And since $a = x^2 + x - 2 \longrightarrow$ then $\frac{da}{dx} = 2x + 1$.

Then, $\frac{dy}{dx} = \frac{dy}{da} \cdot \frac{da}{dx}$

$= 12a^2(2x + 1)$

$= 12(x^2 + x - 2)^2(2x + 1)$

STEPS TO DETERMINE THE DERIVATIVE USING THE CHAIN RULE:

① Identify the inner and outer functions.

② Differentiate outer function; leave the inner function alone.

③ Multiply answer from step 2 by the derivative of the inner function.

EXAMPLES: Differentiate.

1. $f(x) = -3(2x - 3)^{\sqrt{2}}$

2. $f(x) = -\frac{1}{2}(\pi^2 - 2\pi x)^3 + 4\pi x$

3. $h(t) = 4\sqrt{2 + t^2 - 2t^3}$

4. $A(m) = -2\sqrt[3]{m - m^2}$

5. $f(x) = \frac{-1}{(x^2-4)^2}$

6. $y = \frac{2}{2x+5} - 8$

7. $g(x) = \frac{-6}{x^2-2x-8}$

8. $h(n) = \frac{2}{\sqrt{2n^2+4}}$