MHF 4U
COMPOSITION OF FUNCTIONS

Two functions, combined through any of the 4 operations, produce a more complex function. The combined function exhibits properties of each parent function.

If we are given the graph of a combined function,

- properties can be used to identify the original parent functions.
- specific points and features can be used to determine the operation connecting the parent functions.

EXAMPLE: Given $f(x) = x^2 - 4$ and $g(x) = x + 2$,

Then $\boldsymbol{f + g} = (x^2 - 4\,) + (x + 2) = x^2 + x - 2.$

$\boldsymbol{f - g} = (x^2 - 4\,) - (x + 2) = x^2 - x - 6.$

$\boldsymbol{f \cdot g} = (x^2 - 4\,)(x + 2) = x^3 + 2x^2 - 4x - 8 = (x + 2)^2(x - 2).$

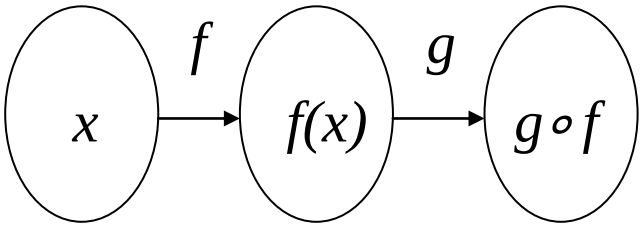
$\frac{\boldsymbol{f}}{\boldsymbol{g}} = \frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2} = x - 2;\ \ x \neq -2.$

The **COMPOSITION** of functions is another form of a combined function.
The composite function requires **substitution** of one function into another, depending on the order of the symbols. The resulting function is also known as a **NESTED** function.

SYMBOLS: Given any 2 functions, $f(x)$ and $g(x)$,

$y = f\big(g(x)\big)$ *or* $y = f \circ g$ *read as "f of g of x"*

$y = g\big(f(x)\big)$ *or* $y = g \circ f$ *read as "g of f of x"*



The values of x are in the domain of $f(x)$ and the values of $f(x)$ are in the domain of $g(x)$.

EXAMPLES:

① Given the graph below, evaluate each of the following:

A) $f \circ g(2)$ B) $g \circ f(2)$ C) $f \circ f(2)$

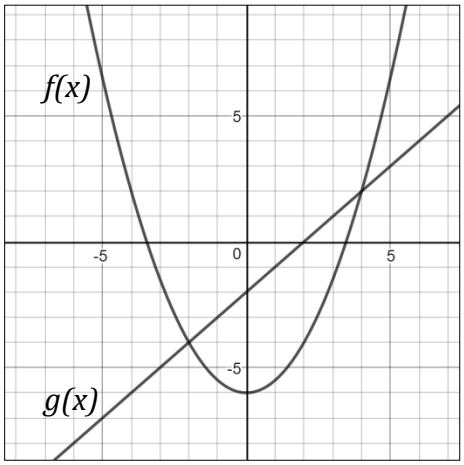
D) $g \circ g(1)$ E) $f \circ g(-3)$ F) $g \circ f(0)$

G) $f \circ g^{-1}(1)$ H) $f^{-1} \circ g^{-1}(-3)$ I) $f^{-1} \circ g(-1)$

J) $f \circ g \circ g(3)$

K) $f = g$

L) $f \circ g = \frac{f}{g}(0)$



② Given $f(x) = x^2 - 2x - 3,\ g(x) = 2x - 1$ and $h(x) = \sqrt{x}$, write an equation for the indicated combination. Then answer the question that follows.

A) $f + g$ B) $g - f$

C) $\frac{f}{g}$ determine the domain D) $f \cdot g$ evaluate $f \cdot g(-1)$

E) $g \circ f$ evaluate $g \circ f = -1$ F) $f \circ g$ evaluate $f \circ g(-2)$

G) $g \circ h$ determine intercepts H) $\frac{g}{f}$ determine the domain

I) $h(g + f)$ determine domain J) $h \circ g \circ f$ determine intercepts

K) $h \circ g$ evaluate $h \circ g = 3$ L) $h \circ f$ determine intercepts