MCR 3U
COMPOUND INTEREST

INVESTMENTS & LOANS

If you invest your money at a financial institution, the bank (or borrower) will secure it and because you are lending your money to the institution, it will also pay you interest on a regular basis. The opposite holds true; that is, if you borrow money from a financial institution, you must repay the full amount to them, plus an interest charge.

Investment: Value of Investment = Amount Invested + Interest Earned

Loan: Total Amount You Owe = Amount of Loan + Interest Charged

There are 3 major factors that determine the accumulated value of your investment or loan...

- the length of time of the investment.
- the interest rate.
- the frequency of interest payments.

Most investments offered by financial institutions provide **COMPOUND INTEREST**.
COMPOUND INTEREST is interest earned on any accumulated money in an account; in other words, the investment earns interest upon interest.

Consider a $100 investment in an account earning 12% per year, compounded monthly. The bank quotes an annual rate, but interest is accumulating on a monthly basis.
12% PER YEAR, compounded annually means 12% interest at the end of 1 year.

12% PER YEAR , compounded monthly means 1% interest at the end of each month for 1 year.

At the end of the first month, the bank will deposit $1 (ie. 1% of $100) into the account.
Then at the end of the next month, the bank will deposit $1.01 (ie. 1% of $101), and so on.

To determine the amount of money that accumulates in an account after a given length of time, the following formula is used:

$$\boldsymbol{A = P(1 + i)^n}$$

Where P = principal amount of money invested
A= accumulated amount of money in the account after n periods
i = interest rate per period
n = number of interest periods

EXERCISE:

1. Determine the interest rate per compounding period given the annual rate.
   A) 12%/a compounded quarterly
   B) 4%/a compounded biweekly
   C) 10%/a compounded bimonthly
   D) 5%/a compounded semiannually

2. How many compounding periods will each investment have?
   A) 5 years, compounded annually
   B) 12 years, compounded semiannually
   C) 20 years, compounded bimonthly
   D) 8 years, compounded quarterly

3. $800 is invested at 8%/a compounded semiannually for 3 years. Find the total amount.

4. Drew would like to invest enough money today so that he will have $4 000 available in 2 years for his first year of university. If his investment well earn 6%/a compounded quarterly, how much must he invest today?

5. How long will it take to double your money if it is invested at 6%/a compounded monthly?

6. Veronika would like to purchase a mini stereo system which costs $1 500. She makes a downpayment of $500 and takes a loan for the rest at 8%/a compounded annually. How much will she have to pay at the end of the 5 year loan? How much interest will she have to pay?

7. Katherine's investment that she made 3 years ago is now worth $5 000. She was earning an interest rate of 10%/a compounded quarterly. How much money did she invest 3 years ago and how much interest did she earn?

8. How long will it take Sabrina to triple her money if she earns 7.25%/a compounded semiannually?

9. Hans won $4 800 in a lottery. He would like to invest the money in one account. One bank has offered him 2 options: a rate of 5%/a compounded quarterly for 5 years or a rate of 5.2%/a compounded semiannually for 5 years. Which option should he choose?

10. Shenae needs $4 000 for her first year of university which starts in 4 years. If she plans to invest $2 500 today and is guaranteed an interest rate compounded semiannually, what annual interest rate should she negotiate?

11. Josh borrowed $10 000 to buy a used car. At the end of his loan period, he was required to pay $3 600 in interest. If the loan was charging him an interest rate of 6%/a compounded biweekly, how long was the loan period?

12. Riley would like to invest enough money today so that she has one million dollars when she retires in 40 years. If she is guaranteed an interest rate of 12%/a compounded semiannually, how much should she invest today? How much interest would she earn on the investment?

13. Solah won $6 000 in a lottery. She decides to invest it for 10 years, earning interest at 6%/a compounded annually. Five years after she started the investment, Solah needed $1 000 to put towards the downpayment of a used car. After withdrawing the money, the bank lowered her interest rate to 5%/a compounded semiannually. How much money is in her account after the 10 year period?

14. Six years before meeting, Josh and Hans each made investments. Josh invested $2 000 earning 8%/a compounded semiannually and Hans invested $1 600 earning 9%/a compounded quarterly. When they met, they decided to consolidate their investments to start a business. How much is their investment worth when they retire in 35 years if their combined account earns a blended rate of 8.5%/a compounded quarterly?

15. Ken's grade eleven class of 12 students got a tip that buying π in Ken's Copasetic Coffee Emporium was a hot deal. Each student bought 30 πs at $2.25/π, then waited until the value of each π reached $10.00 before selling. If the students waited 2 years before selling the π, what was the annual rate of interest earned (compounded monthly) on the π?

16. Sabrina invested $1 000 in an account earning 6%/a compounded semimonthly for 4 years. If Drew waited 2 years, how much would he need to invest in order to accumulate the same amount of money as Sabrina by the same point in time? Drew's account will earn the same rate of interest.

17. Riley buys a new sofa priced at $800. She can pay $800 now or not make any payment now and pay $950 in one year. The salesperson tells Riley that in effect she will have a loan of $800 for one year, compounded monthly. What is the monthly interest rate that Riley would be paying?

18. Katherine buys a stereo. She pays $500 now and agrees to pay $2 000 in three years. If interest on the money is calculated at 16%/a compounded quarterly, determine the value of the stereo now. How much interest would be saves if she paid cash for the stereo when she bought it?

19. A Roman centurion deposited a coin worth 10 cents in a bank at the beginning of the year AD 1.
    A) If the bank paid 3%/a compounded semiannually, what would the investment be worth at the beginning of the year 2000? 4000?
    B) What would the investment be worth in the same years if the interest rate was calculated monthly?

20. New grandparents want to give their grandson a gift on his 18th birthday. They would like to give him the equivalent of what $5 000 would buy on the day he was born.
    A) If inflation is assumed to be 3% per year, how much money should he receive on his 18th birthday?
    B) What is the amount that the grandparents should invest on the day he was born, if the money is invested at 5%/a compounded annually?

21. Veronika purchased a used car for $10 000, plus PST and GST. She gave the dealership a 20% downpayment on the purchase price of the car, then arranged a loan for the remaining amount at 9%/a compounded bimonthly. If Veronika's interest charges totalled $1 200, how long was the period of her loan?

22. A gold bar appreciates in value, on average, 5% per year. Gold costs $552 per ounce and you wish to purchase 10 ounces in 5 years. If you invest $4 000 today in an account earning 6%/a compounded semiannually, will you have enough money to purchase the gold in 5 years?