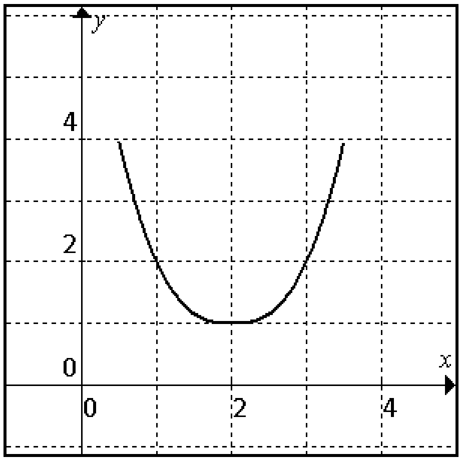
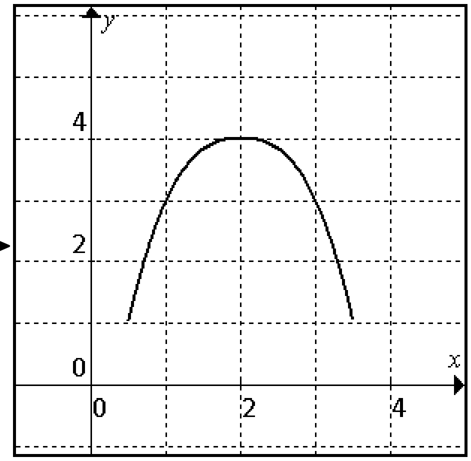
MCV 4U
CONCAVITY & the SECOND DERIVATIVE TEST



When a function lies above all of its tangents, the graph is CONCAVE UP.

When a function lies below all of its tangents, the graph is CONCAVE DOWN.



POINT OF INFLECTION (POI)

The following occur at all POI:

- The graph changes from one concavity to the other.
- $f''(x)$ changes sign.
- $f''(x) = 0 \ \ or \ \ dne$, but this may be true at other points that are not POI.
- A tangent drawn at the point of inflection crosses the curve at that point.

FERMAT'S THEOREM:
All points of inflection have...

- $f'(x) = 0 \ or \ dne$.
- $f''(x) = 0 \ or \ dne$.

BUT the converse is not necessarily true.
That is, if $f''(x) = 0 \ or \ dne$, the point may not be a poi. In this case, the signs of $f''(x)$ to the left and right of the critical point where $f''(x) = 0 \ or \ dne$ are both positive or both negative.

TEST FOR CONCAVITY:

- If $f''(x) > 0$ on an interval from [a, b], then the graph is **CONCAVE UPWARD**.
- If $f''(x) < 0$ on an interval from [a, b], then the graph is **CONCAVE DOWNWARD**.
- Use an **interval chart** to determine concavity by setting $f''(x) = 0$. This is similar to the chart used to determine intervals of increase or decrease.
- Concavity is very helpful in sketching a curve, especially if there are no critical points and the curve is relatively shallow.

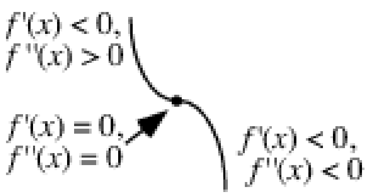
SECOND DERIVATIVE TEST:

- A second method used to determine whether a function has a local maximum, a local minimum, or neither, at a critical number.
- The test does not apply if $f''(c) = 0 \; or \; dne$ (at a cusp, V.A., or other special cases.

Let c be a critical number of a function f.

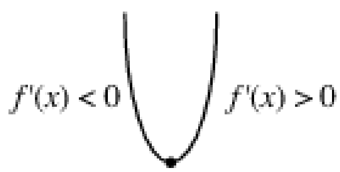
① If $f'(c) = 0$ and $f''(c) > 0$ then f has a **local minimum** at c.

② If $f'(c) = 0$ and $f''(c) < 0$ then f has a **local maximum** at c.



$f''(x) = 0$

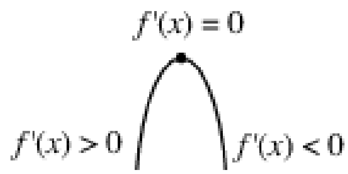
Inflection point



$f'(x) = 0$

$f''(x) > 0$

minimum



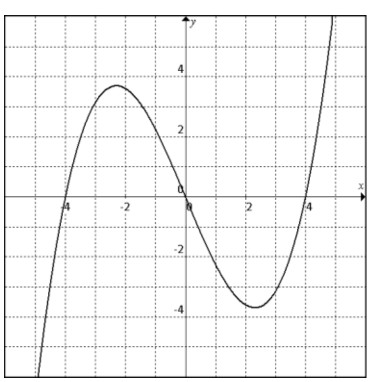
$f'(x) = 0$

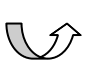
$f''(x) < 0$

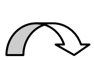
maximum

EXAMPLE ①: State the intervals in which the function is concave up and concave down.

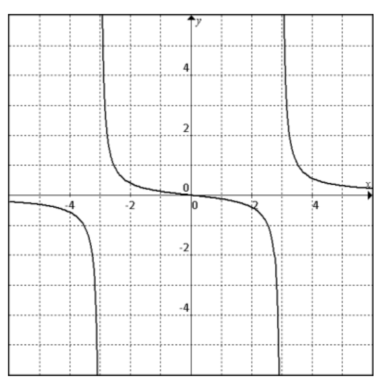
A)

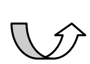


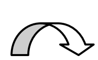




B)







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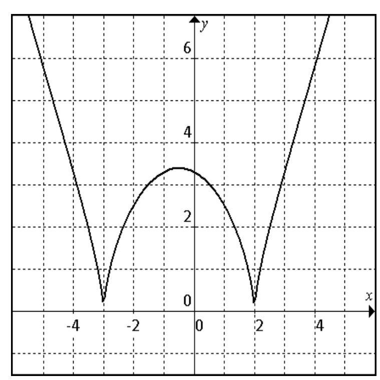
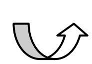
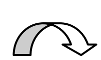






EXAMPLE ②: For each function,

- Determine the critical values.
- Complete an analysis table to determine intervals of ***increase and decrease*** and intervals of ***concavity***.
- Identify the max/min and poi in the analysis table.

A) $f(x) = 4x^3 - 12x^2$

B) $f(x) = \frac{2}{x^2-1}$

C) $f(x) = \frac{2x}{x^2+1}$

D) $f(x) = x^4 - 4x^3 - 18x^2 + 2$

E) $f(x) = (x-2)^{\frac{4}{3}}$

EXAMPLE ③: Use the second derivative test to determine if a critical point corresponds with a local maximum or a local minimum.

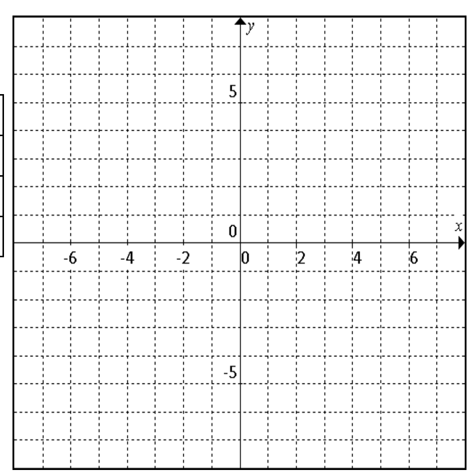
A) $f(x) = x^4 - 8x^2 + 16$

B) $f(x) = \frac{x^3}{x^2-4}$

EXAMPLE ④: Use the limited information to complete each analysis table. Then sketch the function.

A) $f(x) =$ polynomial function

x	(-∞, -3)	(-3, 0)	(0, 2)	(2, 4)	(4, 6)	(6, ∞)
f'(x)	+	-	-	-	-	
f''(x)	-	-	+	-	+	
f(x)						



B) $f(x)$ =rational function with HA at $y = 2$

x	(-∞, -3)	(-3, 0)	(0, 2)	(2, 4)	(4, 6)	(6, ∞)
f'(x)	+	-	-	-	+	+
f''(x)	+	+	-	+	+	-
f(x)						

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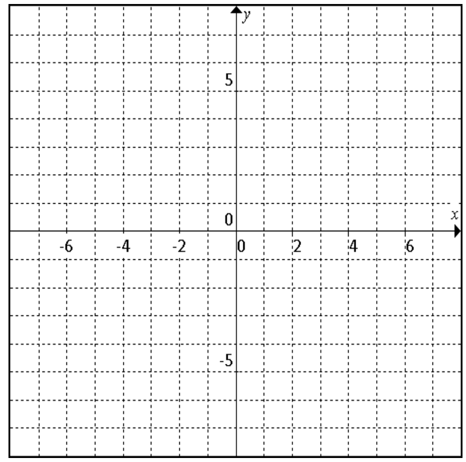

C) $f(x)$ =rational function with...

$\lim_{x \to -2^-} f(x) = \infty$ $\lim_{x \to 2^-} f(x) = -\infty$ $\lim_{x \to \pm\infty} f(x) = 1$

$\lim_{x \to -2^+} f(x) = -\infty$ $\lim_{x \to 2^+} f(x) = -\infty$

x	(-∞, -2)	(-2, 0)	(0, 2)	(2, 4)	(4, 6)	(6, ∞)
f'(x)					-	-
f''(x)					-	+
f(x)