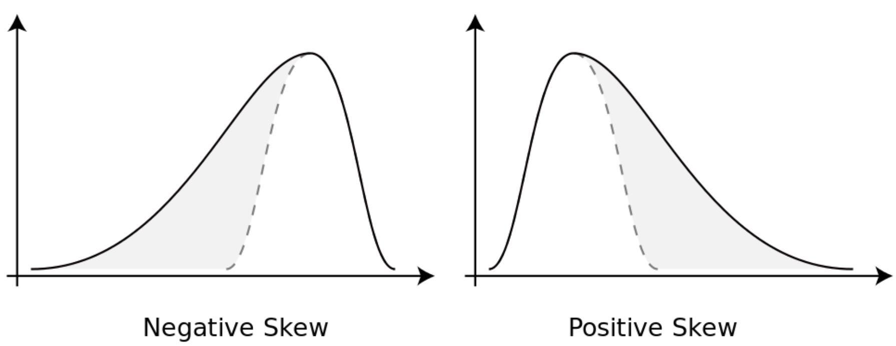
MDM 4U
CONTINUOUS PROBABILITY DISTRIBUTIONS

- Continuous probability distributions allow for fractional or decimal values of the random variable.
- Distributions can be represented by a relative-frequency table, a graph or an equation.
- ***Probabilities*** can be computed by finding the ***area under the curve*** within the appropriate interval.

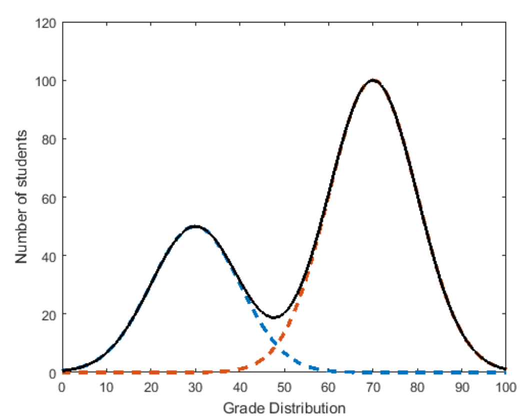
- Continuous probability distributions differ from discrete distributions in that a smooth line or curve is used instead of individual bars representing the discrete values.

SHAPES OF PROBABILITY DISTRIBUTIONS

- A distribution that is not symmetric may be negatively skewed (the mean is to the left of the mode) or positively skewed (the mean is to the right of the mode).

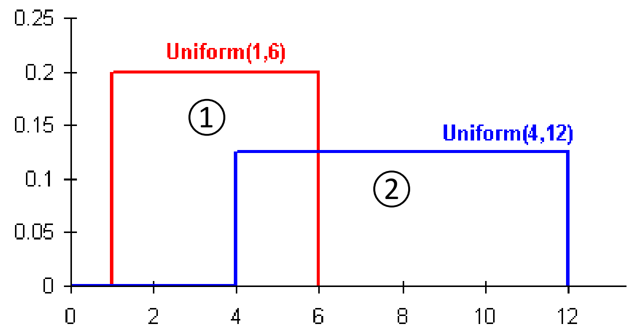


- The above distributions are ***unimodal***, whereas a distribution can be ***bimodal*** as shown to the right.



- ***The probability distribution curve represents the probability density (100% of the data) under the curve.***
- Curves can be modelled by equations which are then used to calculate probabilities.

① UNIFORM DISTRIBUTIONS



- Note that the area under the curve is always equal to 100% or 1.
- The height of the uniform distribution (probability of the continuous random variable) is determined by the area of a rectangle. Since the uniform distribution on the left has a rectangle with base length of 5, and area of a rectangle is $A = LH$, then we have $1 = 5H$, and therefore $H = \frac{1}{5} = 0.2$.
- The probability that a value in this uniform distribution is less than 3, $P(X < 3)$, is determined by calculating the area of the smaller rectangle within that distribution: $P(X < 3) = (3 - 1)(0.2) = 0.4$

To determine the vertical scale of any uniform distribution, we use the formula:

$$\boldsymbol{vertical\ scale} = \frac{\mathbf{1}}{\boldsymbol{x_{max} - x_{min}}}$$

EXAMPLE 1: In the above diagram of the 2 uniform distributions, calculate each probability.

A) For ①:

i) $P(X > 5)$

ii) $P(2 < X < 3.5)$

B) For ②:

i) $P(X < 9.5)$

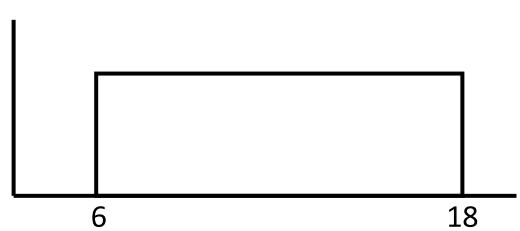
ii) $P(4.7 < X < 8.1)$

EXAMPLE 2: Use the uniform distribution to the right to calculate each probability.

A) $P(X < 12)$

B) $P(X > 15)$

C) $P(10 < X < 14)$



② NON-UNIFORM DISTRIBUTIONS

In the continuous probability distributions below, the variable X is more likely to occur towards the right end of the graph than to the left. To find a probability in a non-uniform distribution, first determine the vertical scale, then find the area inside the shape(s) for the required interval.

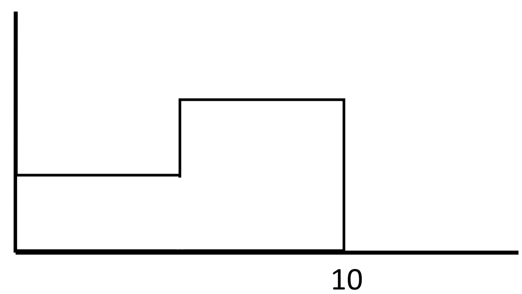
NOTE: The area under any given shape must always equal 1.

EXAMPLE 1: Determine the vertical scale, then calculate each probability.

A) $P(X < 4.5)$

B) $P(X > 2.8)$

C) $P(1.5 < X < 7.5)$

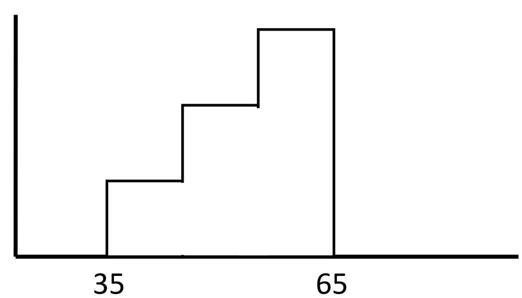


EXAMPLE 2: Determine the vertical scale, then calculate each probability.

A) $P(X < 50)$

B) $P(X > 46)$

C) $P(40 < X < 60)$

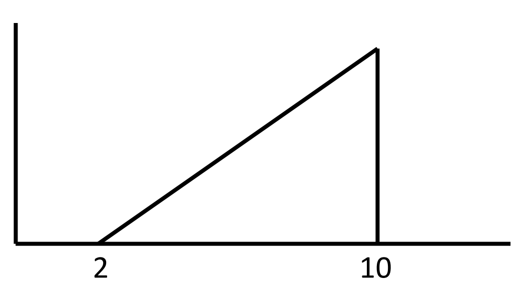


EXAMPLE 3: Determine the vertical scale, then calculate each probability.

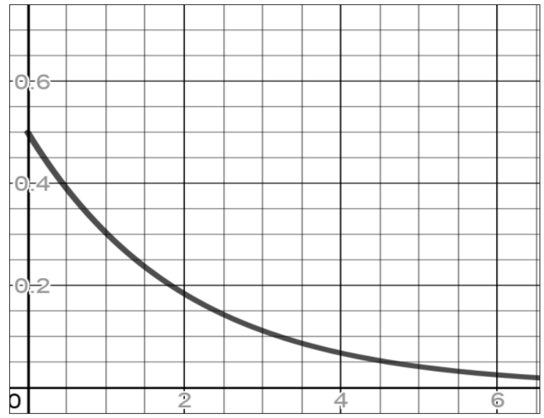
A) $P(X < 7)$

B) $P(X > 4)$

C) $P(3 < X < 6)$



③ EXPONENTIAL DISTRIBUTIONS



- Predicts the waiting times between consecutive events in any random sequence of events.
- The equation for this distribution is $y = ke^{-kx}$ where $k = \frac{1}{\mu}$ is the number of events per unit time and *e*=2.71828.
- The longer the average wait, the smaller the value of *k*, and the more gradually the graph slopes downward.
- Notice that the smallest waiting times are the most likely. This distribution is similar to the discrete geometric distribution in section 7.3. Recall that the geometric distribution models the number of trials before a success. If you think of the event “receiving a phone call in a given minute” as successive trials, you can see that the exponential distribution is the continuous equivalent of the geometric distribution.
- By definition, the exponential distribution predicts the waiting times between consecutive events in any random sequence of events.

Area under the exponential distribution curve is determined using the formula

$$P(X \leq x) = 1 - e^{-kx}$$

Where the "k" value is determined using the μ value (average waiting time) or the y −intercept of the graph.

For the graph above,

- State the value of k.
- Determine each probability:

A) $P(X < 1)$

B) $P(X > 2)$

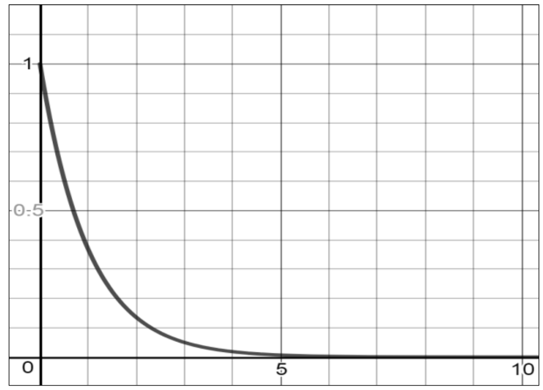
C) $P(1.5 < X < 4)$

EXAMPLE 1: Determine the value of k, then calculate each probability.

A) $P(X < 4.5)$

B) $P(X > 2.8)$

C) $P(1.5 < X < 7.5)$



EXAMPLE 2: Determine the value of k, then calculate each probability.

A) $P(X < 50)$

B) $P(X > 46)$

C) $P(40 < X < 60)$