MCV 4U

CROSS PRODUCT OF 2 VECTORS

① The **cross product** of 2 vectors, $\vec{a} = (x_1,\ y_1,\ z_1)$ and $\vec{b} = (x_2,\ y_2,\ z_2)$, is a third vector which is perpendicular to the 2 vectors, and is determined by the following formula:

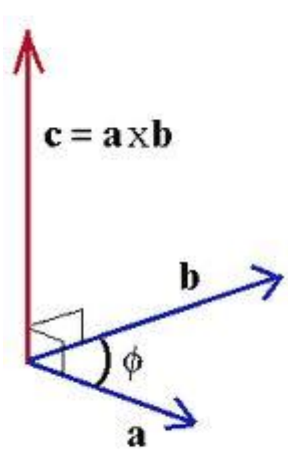
$$\vec{a} \times \vec{b} = \left(\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}, \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}, \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \right)$$

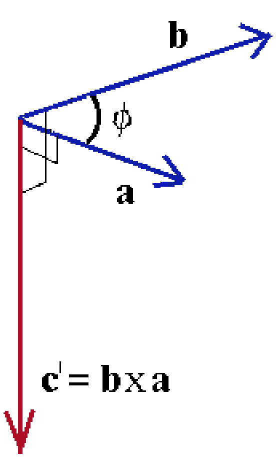
where $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

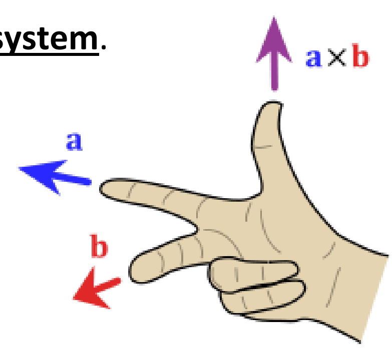
② If $\vec{v} = \vec{a} \times \vec{b}$, then... $\vec{v} \cdot \vec{a} = 0$ and $\vec{v} \cdot \vec{b} = 0$
which follows from Section 5.3

③ $\left|\vec{a} \times \vec{b}\right| = |\vec{a}|\left|\vec{b}\right| \sin\theta;$ θ= angle between the 2 vectors

④ All 3 vectors $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ form a **right-handed system**.







EXAMPLES:

1. Find a vector, $\vec{v}$, that is perpendicular to both (1, 3, 2) and (4, -6, 7).
   Show that $\vec{v}$ is perpendicular to each vector.

2. Find a unit vector perpendicular to $\vec{a} = \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$

3. Show that $\hat{\imath} \times \hat{k} = -\hat{\jmath}$. [Convert the unit vectors to algebraic vectors.]

4. Given $\vec{u} = (2, 1, 0)$, $\vec{v} = (-2, 3, 1)$ and $\vec{w} = (1, -4, -2)$ calculate each of the following:
   [Explain why the cross product in parts A and B must be done before the dot product.]

   A) $\vec{u} \times \vec{w} \cdot \vec{v}$ B) $\vec{u} \times \vec{v} \cdot \vec{w}$ C) $\vec{u} \times (\vec{w} \times \vec{v})$