MCV 4U

PART 1: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS (using First Principles)

① **Derivative of $f(x) = \sin x$** using first principles.

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x(\cos h - 1) + \cos x\sin h}{h}$$

$$= \lim_{h\to 0}\frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0}\frac{\cos x\sin h}{h}$$

$$= \sin x\lim_{h\to 0}\frac{\cos h - 1}{h} + \cos x\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \sin x\,(0) + \cos x\,(1)$$

$$\boxed{\frac{d}{dx}\sin x = \cos x}$$

② **Derivative of $f(x) = \cos x$** using first principles.

$$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x(\cos h - 1) - \sin x\sin h}{h}$$

$$= \lim_{h\to 0}\frac{\cos x(\cos h - 1)}{h} - \lim_{h\to 0}\frac{\sin x\sin h}{h}$$

$$= \cos x\lim_{h\to 0}\frac{\cos h - 1}{h} - \sin x\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \cos x\,(0) - \sin x\,(1)$$

$$\boxed{\frac{d}{dx}\cos x = -\sin x}$$

PART 2: DERIVATIVES OF EXPONENTIAL FUNCTIONS (using First Principles)

① **Derivative of** $\boldsymbol{f(x) = 4^x}$ using first principles.

$$\frac{d}{dx}\,4^x = \lim_{h\to 0}\frac{4^{x+h} - 4^x}{h}$$

$$= \lim_{h\to 0}\frac{4^x(4^h - 1)}{h}$$

$$= 4^x \lim_{h\to 0}\frac{4^h - 1}{h}$$

x	$\frac{4^h-1}{h}$
0.00001	
-0.00001	
average	
ln 4	

② **Derivative of** $\boldsymbol{f(x) = 10^x}$ using first principles.

$$\frac{d}{dx}\,10^x = \lim_{h\to 0}\frac{10^{x+h} - 10^x}{h}$$

$$= \lim_{h\to 0}\frac{10^x(10^h - 1)}{h}$$

$$= 10^x \lim_{h\to 0}\frac{10^h - 1}{h}$$

x	$\frac{10^h-1}{h}$
0.00001	
-0.00001	
average	
ln 10	

③ **Derivative of** $\boldsymbol{f(x) = e^x}$ using first principles.

$$\frac{d}{dx}\,e^x = \lim_{h\to 0}\frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h\to 0}\frac{e^x(e^h - 1)}{h}$$

$$= e^x \lim_{h\to 0}\frac{e^h - 1}{h}$$

x	$\frac{e^h-1}{h}$
0.00001	
-0.00001	
average	
ln e	

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS:

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$

$\frac{d}{dx}\sec x = \sec x \cdot \tan x$

$\frac{d}{dx}\cot x = -\csc^2 x$

DERIVATIVES OF EXPONENTIAL FUNCTIONS:

$\frac{d}{dx}a^x = a^x \ln a$

$\frac{d}{dx}e^x = e^x$

DERIVATIVES OF LOGARITHMIC FUNCTIONS:

$\frac{d}{dx}\log_{\mathrm{a}} x = \frac{1}{x \ln a}$

$\frac{d}{dx}\ln x = \frac{1}{x}$