MCV 4U

THE DOT PRODUCT

①. Dot Product of $\vec{u}$ *and* $\vec{v}$ is defined as follows:

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|cos\theta}$$

where θ = angle between the vectors

It follows that $\vec{u} \cdot \vec{u} = |\vec{u}||\vec{u}|cos0° = |\vec{u}|^2$

②. If $\vec{u}$ and $\vec{v}$ are non-zero vectors and $\vec{u} \cdot \vec{v} = 0$, then the vectors are $\perp$.

Proof:

If $\vec{u} \cdot \vec{v} = 0$,

Then $|\vec{u}||\vec{v}|cos\theta = 0$

Since $\vec{u}$ and $\vec{v}$ are non-zero vectors,

Then it follows that $cos\theta = 0$

Therefore, $\theta = cos^{-1}(0) = \pm 90°$

③. As a result of #1 and #2 above, the following properties exist:

$\hat{\imath} \cdot \hat{\imath} = |\hat{\imath}|^2 = 1$

$\hat{\jmath} \cdot \hat{\jmath} = 1$

$\hat{k} \cdot \hat{k} = 1$

$\hat{\imath} \cdot \hat{\jmath} = 0$

$\hat{\imath} \cdot \hat{k} = 0$

$\hat{\jmath} \cdot \hat{k} = 0$

④. Using the properties established in #3 above, we can show an alternative calculation for the dot product.

$$\vec{u} \cdot \vec{v} = (u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}) \cdot (v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k})$$

$$= u_xv_x\hat{\imath}\hat{\imath} + u_xv_y\hat{\imath}\hat{\jmath} + u_xv_z\hat{\imath}\hat{k}$$
$$+ u_yv_x\hat{\jmath}\hat{\imath} + u_yv_y\hat{\jmath}\hat{\jmath} + u_yv_z\hat{\jmath}\hat{k}$$
$$+ u_zv_x\hat{k}\hat{\imath} + u_zv_y\hat{k}\hat{\jmath} + u_zv_z\hat{k}\hat{k}$$

$$\boxed{\vec{u} \cdot \vec{v} = u_xv_x + u_yv_y + u_zv_z}$$

EXAMPLES:

1. Determine the dot product of each pair of vectors.

   A) $|\vec{u}| = 6, |\vec{v}| = 4, \theta = 40°$ B) $\vec{u} = (2, -2,5), \vec{v} = (-1, 6, 3)$

2. Determine 2 vectors that are perpendicular to $\vec{a} = (4, -6)$.

3. Determine the angle between $\vec{u} = (2, -2,5)$ and $\vec{v} = (-1, 2, 3)$.

4. Given $\vec{u} = (-2,\ n,\ 6)$ and $\vec{v} = (1, 4, -3)$, determine the value of n if...

   A) the vectors are collinear. B) the vectors are perpendicular.

5. Find a vector that is perpendicular to both $\vec{u} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{v} = -2\hat{\jmath} - 4\hat{k}$.

6. Expand and simplify $(3\vec{a} + \vec{b}) \cdot (6\vec{b} - 3\vec{a})$.

7. Expand $(2\hat{\imath} - 3\hat{k}) \cdot (\hat{\imath} + 4\hat{k})$.

8. $\hat{u}$ and $\hat{v}$ are unit vectors at 60° to each other, calculate $(5\hat{u} + \hat{v}) \cdot (\hat{u} - \widehat{2v})$.