MHF 4U
CHARACTERISTICS of POLYNOMIAL FUNCTIONS in FACTORED FORM

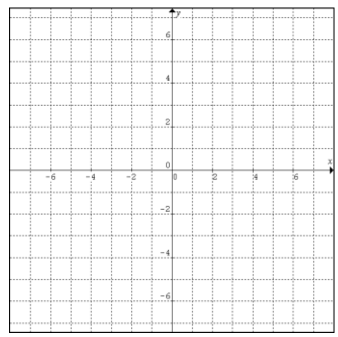
Consider the function $f(x) = x^3 - 2x^2 - 4x + 8$. Without sketching its graph, state its...

Degree	
Max # of turning points	
Max # of zeros	
Y-intercept	
End behaviour	

EXAMPLES: Consider the polynomial functions. State the indicated properties.

① $f(x) = (x-3)(x+2)$

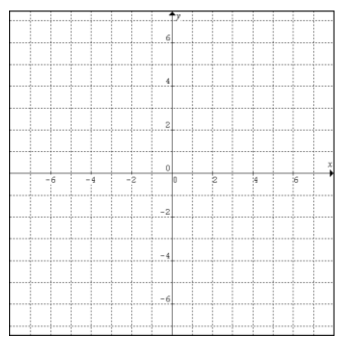
degree =
number of turning points =
y –intercept =
zeros =
end behaviour =



Comments:

② $f(x) = -(x-2)^2$

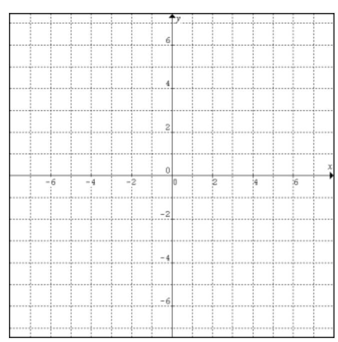
degree =
number of turning points =
y –intercept =
zeros =
end behaviour =



Comments:

③ $f(x) = x^2 + 1$

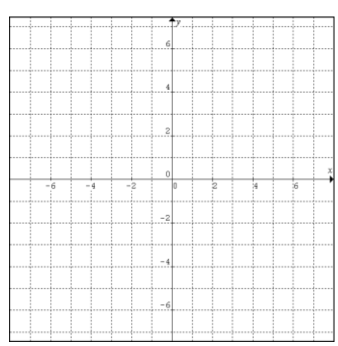
degree =
number of turning points =
y –intercept =
zeros =
end behaviour =



Comments:

④ $f(x) = -x^2 - 3x + 4$

degree =
number of turning points =
y –intercept =
zeros =
end behaviour =



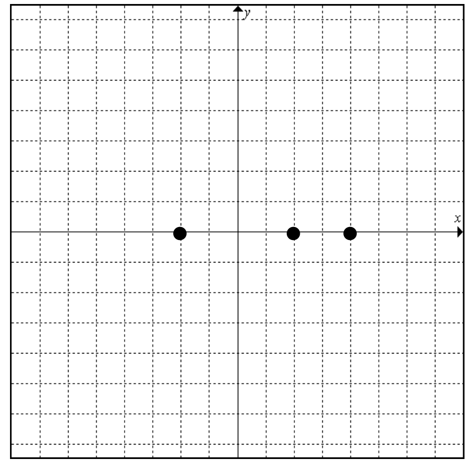
Comments:

Consider the function $f(x) = (x-2)(x+2)^2(x-4)$.

The zeros (or x −intercepts) are located at $x = 2, -2, 4$, determined by solving the equation $f(x) = 0$.

The zeros of $f(x)$ are shown in the graph to the right.

Complete the curve of $f(x)$.



ANALYSIS TABLES

Using properties of the equation of $f(x)$, the shape of the graph can be determined as it curves through the zeros; that is, we can find if $f(x)$ approaches the first x −intercept from above or below the x −axis.

Analyze the ***domain intervals surrounding the x −intercepts***.

① Prepare intervals of the domain separated by the zeros.

(-∞,-2) (-2,2) (2,4) (4,∞)

② Choose test values within each interval. Use the test values in the factored form of f(x). If the overall sign of f(x) is positive, then the function is above the x-axis in that domain interval; if the sign of f(x) is negative, then it is below the x-axis in that domain interval.

domain	(-∞,-2)	(-2,2)	(2,4)	(4,∞)
Test value	-3	0	3	5
$(x-2)$ **factor**	-	-	+	+
$(x+2)$ **factor**	+	+	+	+
$(x-4)$ **factor**	-	-	-	+
Overall $f(x)$				

③ The ***overall signs of f(x)*** determined in step 2 allow us to sketch *f(x)* passing through the zeros. Take note of any special features, such as a factor with an order of 2 or 3.