MCR 3U

PART 1: GEOMETRIC SEQUENCES

Geometric sequences: sequences in which successive terms are determined by multiplying or dividing the same value.

Examples:

1. 6, 12, 24, 48, ...
2. 400, -200, 100, -50, ...
3. 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, ...
4. $2x$, $6x^2$, $18x^3$, ...

In any sequence,

$t_1 = first\ term$

$t_2 = second\ term$

...

$t_n = nth\ term\ or\ general\ term\ or\ general\ formula$

EXAMPLE 1: Given $t_n = 3\left(\frac{1}{2}\right)^{n+2}$, find the first 3 terms.

EXAMPLE 2: Given $t_n = 2(1.2)^{n+1}$, find the fifth and twentieth terms.

In general, if the "multiplier" between terms is represented by the symbol "***r***" for ***ratio***, the general formula for a geometric sequence is

$$\boxed{\boldsymbol{t_n = t_1 r^{n-1}}}$$

where n = number of terms or the position of a term
r = common ratio or multiplier

PROOF:

The general geometric sequence is...

EXAMPLE 3: Given the sequence of terms ¼, 2, 16, 128, ... find the general formula. Find t_{10}.

EXAMPLE 4: How many terms are in the sequence ½ , 2, 8, ..., 8192?

EXAMPLE 5: In a geometric sequence, the fifth term is 1536 and the ninth term is 24576. Find the first term and the common ratio. Then determine the general formula for the sequence.

PART 2: GEOMETRIC SERIES

$2, 6, 18, 54, \ldots$ is called an Geometric Sequence, since it has a common ratio of $\frac{6}{2} = 3$.

A ***GEOMETRIC SERIES*** is the sum of the terms of a geometric sequence.

In the sequence above, $2 + 6 + 18 + 54 + \cdots$ would represent the geometric series.

As in an Arithmetic Series, the symbol $\boldsymbol{S_n}$ represents the **SUM of the first $\boldsymbol{n}$ terms** of a geometric series. For the above series, $S_5 = 2 + 6 + 18 + 54 + 162 = 242$.

Using general symbols, the formula for S_n of a geometric series is developed.

① $S_n = t_1 + (t_1 r) + (t_1 r^2) + (t_1 r^3) + \cdots + (t_1 r^{n-2}) + (t_1 r^{n-1})$

② $rS_n = t_1 r + (t_1 r^2) + (t_1 r^3) + (t_1 r^4) + \cdots + (t_1 r^{n-1}) + (t_1 r^n)$

①−② $S_n - rS_n = t_1 - t_1 r^n$

Factoring S_n: $S_n(1 - r) = t_1(1 - r^n)$

$$\boxed{S_n = \frac{t_1(1 - r^n)}{1 - r}} \quad \textbf{OR} \quad \boxed{S_n = \frac{t_1(r^n - 1)}{r - 1}}$$

EXAMPLE 1: Find S_{10} for the series defined by $t_n = 2^{n-1}$.

EXAMPLE 2: Find the sum of the series $59049 + 19683 + \cdots + 9$.

EXAMPLE 3: Find the sum of the first 5 terms of the series $100 + 25 + 6.25 + \ldots$

EXAMPLE 4: Gerry earned a salary of $25 000 in his first year as an employee of WallyMart. Each year, his annual salary increases by 5% of the previous year. What is Gerry's overall earnings for his first 10 years working at WallyMart?