MCV 4U
INTERSECTION OF A LINE AND A PLANE

① POINT OF INTERSECTION (POI) BETWEEN LINE & PLANE

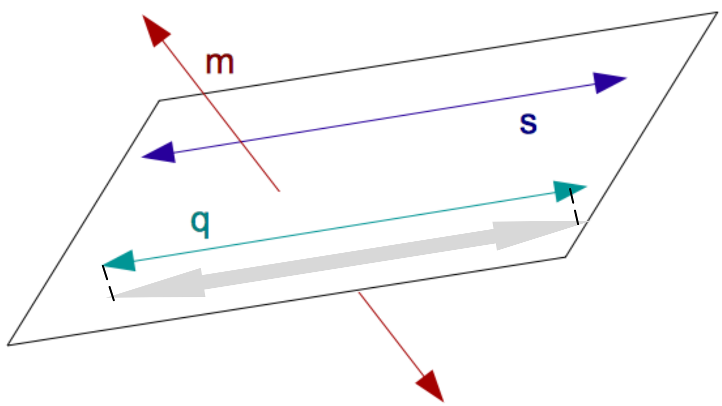
GIVEN:
- Equation of Line in any form – convert to PARAMETRIC equations
- SCALAR Equation of Plane

SOLUTION:
- Substitute parametric equations of line into scalar equation of plane and solve for t.

- <u>**Three possibilities**</u>:

$t = \textbf{real number}$	$0t = \textbf{real number}$	$0t = 0$
• POI exists • substitute t into parametric equation to find POI. • LINE **m** in diagram	• t is undefined • no POI • line is parallel to plane • LINE **q** in diagram	• t can be any value • line lies in the plane • LINE **s** in diagram

NOTE: If the $\vec{d}(of\ line) \cdot \vec{n}(of\ plane) = 0$, (ie. dot product), then the line is parallel to the plane. And if a point of the line lies in the plane, then the line is coincident with the plane.



EXAMPLE ①: Find the poi, if it exists. Otherwise, describe the nature of the line and the plane.

A) $2x + y - 4z - 8 = 0$ and $\vec{r} = (1, -8, 7) + t(1, 3, -4)$

B) $\vec{r_1} = (-3, 3, -4) + t(4, -2, 3)$ and $\vec{r_2} = (1, 1, -1) + s(1, 1, -3) + u(2, 0, -1)$

C) $\frac{x+3}{2} = y - 5 = -\frac{1}{2}z + 1$ and $\vec{r_2} = (4, 2, -1) + s(0, 1, 1) + u(2, 2, -1)$

② FINDING THE POI BETWEEN A PLANE & COORDINATE AXES

NOTE: $x-, y-,$ and z-intercepts are key to making sketches of planes in 3 D.

(A) GIVEN:

- SCALAR equation of a plane.

SOLUTION:

- Find x-intercept by setting $y = z = 0$ and solve for x.
- Use same method to find the $y-$ and $z-$intercepts.

- If normal to the plane has one zero component, then the plane is parallel to the axis represented by that component. For example, the plane $2x - 3z + 5 = 0$ has normal $(2, 0, -3)$ and is parallel to the y-axis.

- If D (in the scalar equation) is zero, then the plane contains the origin. For example, the plane $3x - y + 4z = 0$ passes through the origin.

(B) GIVEN:

- PARAMETRIC equations of a plane

SOLUTION:

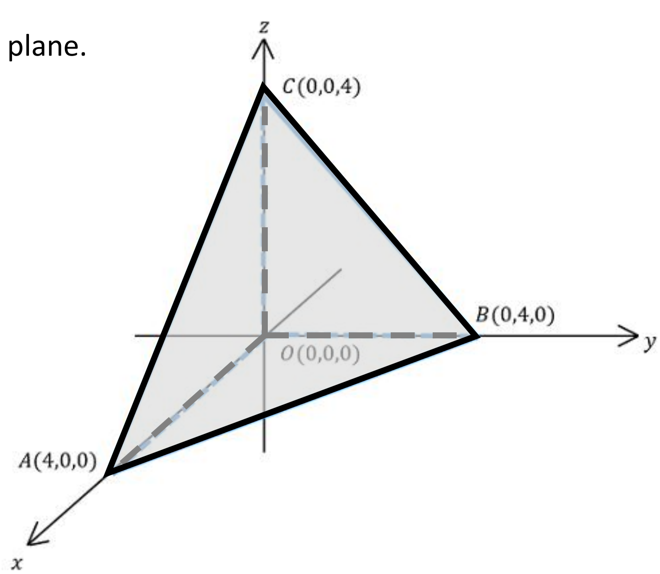
- Find x-intercept by setting $y = 0$ and $z = 0$ and solving 2 parametric equations for s and t by substitution or elimination.
- Substitute s and t into x parametric equation to find x-intercept.
- Use the same method to find $y-$ and z-intercepts.

EXAMPLE ②: Determine the intercepts of each plane.

A) $2x + 4y - z - 8 = 0$

B) $\vec{r} = (2, 3, 6) + s(2, 2, -1) + t(0, 1, 1)$

C) $x + y + z - 4 = 0$



③ FINDING THE LINE OF INTERSECTION (LOI) BETWEEN A PLANE & A COORDINATE PLANE

(A) GIVEN:

- PARAMETRIC equations of a plane

SOLUTION:

- Find xy −intercept, set $z = 0$ and solve for t in terms of s.
- Substitute t into the other 2 parametric equations of the plane and simplify.
- The intersection of the plane with the xy −plane is the line given by the simplified parametric equations of x and y.

(B) GIVEN:

- SCALAR equation of a plane

SOLUTION:

- Find xy −intercept, set $z = 0$ and simplify.
- What is left is a scalar equation of the line of intersection in the xy −plane.
- Use the same method to find the yz − and xz −intercepts.

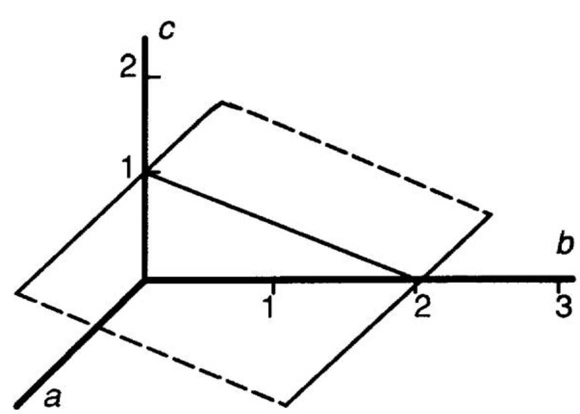
EXAMPLE ③: Find the LOI of each plane with the coordinate planes.

A) $y + 2z - 2 = 0$

B) $\vec{r} = (1, 1, -2) + t(3, -1, 2) + u(1, -1, -2)$

C) $3x - 2y + 2z - 12 = 0$

D) $x - 3y + 6 = 0$

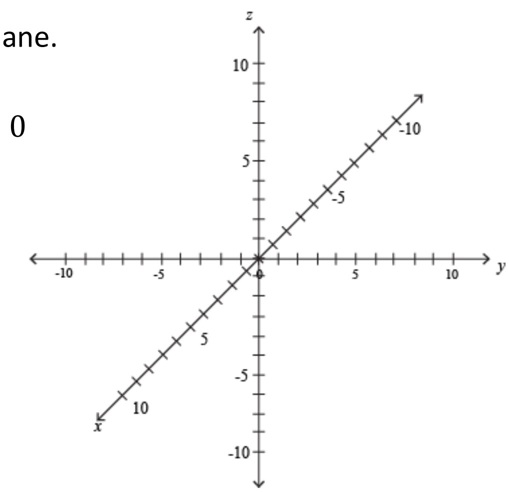


④ SKETCHING PLANES IN 3-SPACE

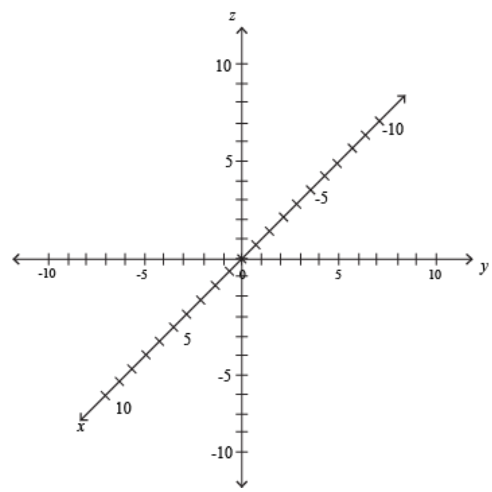
- Determine the POIs of the plane with the coordinate axes
  AND/OR
- Determine the LOIs of the plane with the coordinate planes

EXAMPLE ④: Sketch each plane.

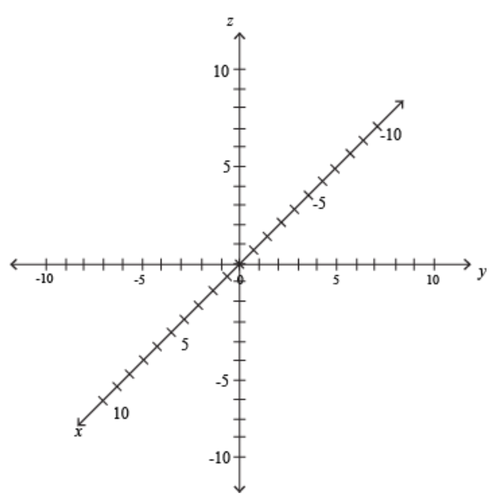
A) $3x - 8y - 8z + 24 = 0$



B) $3x + 2y - 18 = 0$



C) $4x + 12 = 0$



D) $5x - 3y = 0$