MCV 4U

EQUATIONS OF A LINE IN R^3

① **VECTOR EQUATION**:

$$\overrightarrow{OP} = \overrightarrow{OP_O} + t\vec{m}$$
$$\vec{r} = \overrightarrow{r_O} + t\vec{m}$$
$$(x, y, z) = (x_o,\ y_o, z_o) + t(a, b, c)$$

(a, b, c) = direction numbers of the line typically written as integers.

② **PARAMETRIC EQUATION**:

$$x = x_o + at$$
$$y = y_o + bt$$
$$z = z_o + ct$$

③ **SYMMETRIC EQUATION**:

$$\frac{x - x_o}{a} = \frac{y - y_o}{b} = \frac{z - z_o}{c}$$

NOTE: If a component of the direction vector is 0, then that component is written separate from the symmetric equation. For instance, if $b = 0$, then the symmetric is written as $\frac{x - x_o}{a} = \frac{z - z_o}{c}; y = y_o$

EXAMPLE ① Write the vector, parametric and symmetric equations for the line passing through $A(2, -2, -8)$ and $B(5, -2, -14)$

$\vec{m} = \overrightarrow{AB} =$

VECTOR:

PARAMETRIC:

SYMMETRIC:

EXAMPLE ② Given $x - 4 = \frac{y+2}{3} = z$, write a vector equation for the line.

The NATURE of 2 LINES IN R^3 may be described as...

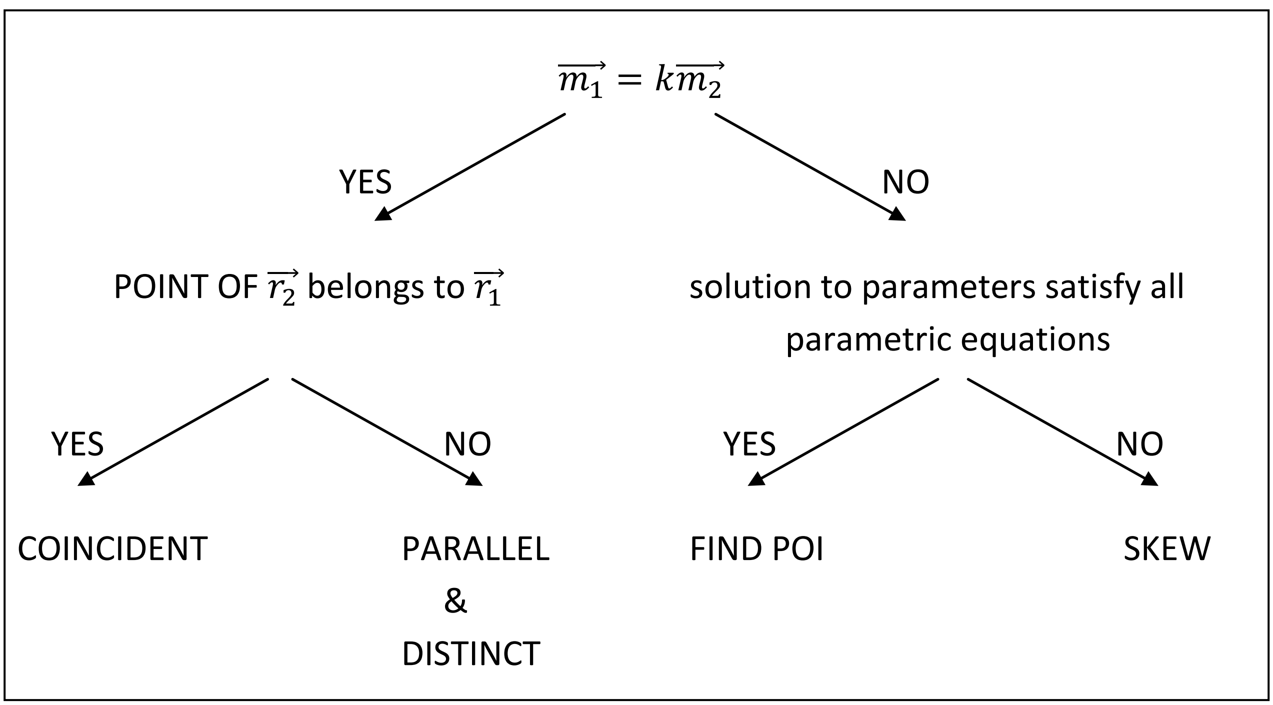
① **COINCIDENT:** same line, so infinite number of common points

② **PARALLEL & DISTINCT**: 2 separate lines; no point of intersection

③ **INTERSECT AT A POINT**: find the point of intersection

④ **SKEW:** lines that are not parallel and do not intersect
-- lie in different planes

In order to determine how 2 lines are related, use the flow-chart below:



EXAMPLE ③ Given $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$, determine the nature of the 2 lines.

$$\overrightarrow{r_1}: \frac{x-5}{2} = \frac{y+4}{-5} = \frac{z+1}{3} \qquad \overrightarrow{r_2}: \frac{x+1}{-4} = \frac{y-11}{10} = \frac{z+4}{-6}$$

STEP 1: **Check for parallel lines.**

STEP 2: **Check if a point of one line belongs to the other line. If "t" values are equivalent for all components, then the lines are __________________.**

EXAMPLE ④ $\overrightarrow{r_1} = (-1, 1, 0) + t(3, 4, -2)$ and $\overrightarrow{r_2} = (-1, 0, -7) + s(2, 3, 1)$

STEP 1: **Check for parallel lines.**

STEP 2: **Equate the parametric equations of the first line with the parametric equations of the second line. If the parameters "t" and "s" satisfy all 3 equations, then the lines are __________________.**