

MHF 4U

SOLVING LOGARITHMIC EQUATIONS

Solving logarithmic equations requires one of the following 2 cases:

CASE 1: *Single logarithm on both sides of the equation.*

$$\log_a(f(x)) = \log_a(g(x)) \quad \dots \text{if bases are equal, equate the arguments.}$$

$$f(x) = g(x) \quad \dots \text{then solve for } x.$$

CASE 2: *Single logarithm on one side of the equation.*

$$\log_a(f(x)) = n \quad \dots \text{convert to exponential form.}$$

$$a^n = f(x) \quad \dots \text{then solve for } x.$$

EXERCISE: Solve each logarithmic equation. Check all solutions.

1. $\log_x \frac{1}{8} = 2$

2. $\log_x \frac{1}{27} = -2$

3. $\log x = 2 - \log 5$

4. $2\log x = \log(5x - 4)$

5. $\log x = 2 - 2\log 2$

6. $\log_{x-1}(x^2 - 5x + 10) = 2$

7. $2\log x = -\log(6 - x^2)$

8. $\log\log\log x = 0$

9. $\log_2 \log_3 \log_4 x = 0$

10. $\log\left(3^{\sqrt{\frac{x(x-4)}{x-3}}} + 1\right) = 1$

11. $\log(2x - 5) + \log(3x + 1) = 1$

12. $\log_3(x + 2) + \log_3(4x - 1) = 2$

13. $\log_5 x + 1 = \log_5(4x + 5)$

14. $\log^2(x + 5) - 3\log(x + 5) = 0$

15. $2\log_4^2(x - 1) - \log_4(x - 1) = 1$

16. $5\log_3(x + 2) - 6\log_3^2(x + 2) = -14$

Answers: 1) $\frac{1}{2\sqrt{2}}$; 2) $3\sqrt{3}$; 3) 20; 4) 4; 5) 25; 6) 3; 7) $\sqrt{3}$; 8) 10^{10} ; 9) 64; 10) 2, 6;

11) 3; 12) 1; 13) 5; 14) 995, -4; 15) 1.5, 5; 16) 7, $-2 + 3^{-7/6}$