

MHF 4U

LAWS OF LOGARITHMS

RECALL Laws of Exponents:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$a^0 = 1$$

LAWS OF LOGARITHMS:

$$\log_a m + \log_a n = \log_a(mn)$$

$$\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$$

$$n \log_a m = \log_a m^n$$

$$\log_a 1 = 0$$

EXERCISE: Use the laws of logarithms for each of the following questions.

1. Express as a single logarithm.

A) $2\log a + 4\log b$

B) $2\log_3 k - \frac{1}{2}\log_3 w - 3\log_3 t$

C) $3\ln 2 - 2\ln h + \frac{1}{3}\ln k$

D) $-2\log_2 p - \frac{3}{2}\log_2 d$

2. Expand each logarithm.

A) $\log_2(a^2 bc^3)$

B) $\log_5\left(\frac{3a}{5bc^2}\right)$

C) $\ln\left(\frac{6e^2}{\sqrt{h}}\right)$

D) $\ln\left(\frac{1}{4\sqrt[3]{e}}\right)$

3. Simplify each of the following. Then evaluate.

A) $\log_3\left(\frac{1}{81}\right)$

B) $\log_6 \sqrt[3]{\frac{1}{36}}$

C) $\ln\left(\frac{1}{\sqrt[5]{e^2}}\right)$

D) $\log_2 25.6 + \log_2 5$

4. Given $\log_a 3 = k$ and $\log_a 5 = d$, use the laws of logarithms to rewrite each expression:

A) $\log_a 15$

B) $\log_a 0.6$

C) $\log_a 9$

D) $\log_a 0.2$

E) $\log_a 3a$

F) $\log_a (25a^3)$

G) $\log_a 0.04a$

H) $\log_a 5.4a^2$

5. Rewrite each logarithmic function using the laws of logarithms.

A) $f(x) = \log (1000x)$

B) $y = \log_2 64x^2$

C) $g(x) = \log_4 \frac{1}{64\sqrt{x}}$

D) $P(t) = \ln \left(\frac{1}{e^2 t^3} \right)$

E) $H(t) = \ln(\sqrt{et^3})$

F) $a(n) = \log_2 \left(\frac{1}{32n} \right)$

Answers:

1. a) $\log[a^2b^4]$; b) $\log[k^2/(w^{1/2}t^3)]$; c) $\ln[8k^{1/3}/h^2]$;
2. a) $2\log a + \log b + 3\log c$; b) $\log 3 + \log a - \log 5 - \log b - 2\log c$; c) $\ln 6 + 2 - \frac{1}{2}\ln h$;
3. a) -4; b) -2/3; c) -2/5; d) 7;
4. a) kd ; b) k/d ; c) $2k$; d) $-d$; e) $d+1$; f) $2d+3$; g) $-2d+1$; h) $3k-d+2$;
5. a) $\log x + 3$; b) $2\log x + 6$; c) $-1/2\log x - 3$; d) $-3\ln t - 2$; e) $3/2\ln t + 1/2$; f) $-\log n - 5$