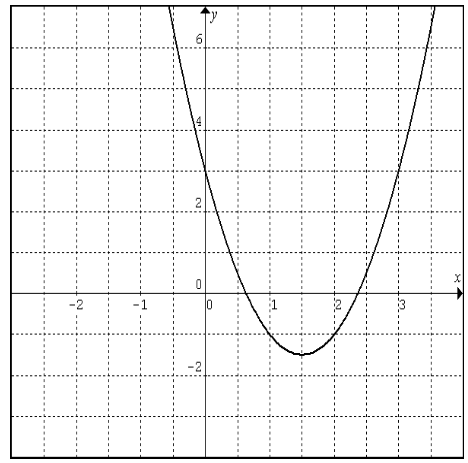
MCR 3U

DETERMINING MAX/MIN VALUES OF QUADRATIC FUNCTIONS

PART A: USING THE Y-INTERCEPT METHOD

Consider the quadratic function $f(x) = 2x^2 - 6x + 3$.

Its graph is shown to the right.



A) Highlight the y-intercept. How does the y-intercept relate to the equation of the function?

B) Identify another point on $f(x)$ that is symmetric with the y-intercept. State the coordinates of this point.

C) How can we use the y-intercept and the point in part B to find the ***axis of symmetry***? Find its location algebraically.

D) How can you use the value of the axis of symmetry to find the vertex? Find its location algebraically. State the coordinates of this point, then state whether it is a ***maximum or a minimum point***.

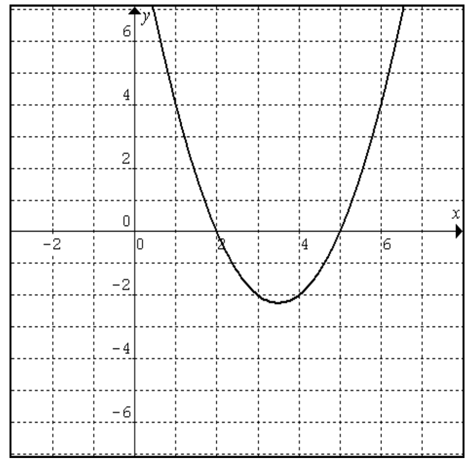
EXAMPLE 1: $f(x) = \frac{1}{2}x^2 - 2x - 4$

EXAMPLE 2: $f(x) = -\frac{2}{3}x^2 + x + 1$

PART B: USING THE X-INTERCEPTS METHOD

Consider the quadratic function $f(x) = (x-2)(x-5)$.

Its graph is shown to the right.



A) Highlight the x-intercepts. How do the x-intercepts relate to the equation of the function?

B) How can you use the x-intercepts to find the ***axis of symmetry***? Find its location algebraically.

C) Find the vertex algebraically. State the coordinates of this point, then state whether it is a ***maximum or a minimum point***.

EXAMPLE 1: $f(x) = \frac{1}{2}x^2 - x - 4$

EXAMPLE 2: $f(x) = -\frac{2}{3}x^2 + \frac{1}{3}x + 2$

PART C: USING THE VERTEX METHOD

$$y = ax^2 + bx + c \longrightarrow y = a(x-p)^2 + q$$

Complete the square

TO COMPLETE THE SQUARE	EXAMPLE:
1. Order terms; group x terms together using [ ].	$y = 1 - 3x + 4x^2$ $y = [4x^2 - 3x] + 1$
2. Factor "a" value from [ ].	$y = 4\left[x^2 - \frac{3}{4}x\right] + 1$
3. Find missing value that "completes the square" with x terms. Add and subtract this value inside the [ ].	$\left[\frac{1}{2}\left(-\frac{3}{4}\right)\right]^2 = \frac{9}{64}$ $y = 4\left[\left(x^2 - \frac{3}{4}x + \frac{9}{64}\right) - \frac{9}{64}\right] + 1$
4. Rewrite the trinomial into its perfect square binomial.	$y = 4\left[\left(x - \frac{3}{8}\right)^2 - \frac{9}{64}\right] + 1$
5. Expand the "a" value through [ ].	$y = 4\left(x - \frac{3}{8}\right)^2 - \frac{9}{16} + 1$
6. Collect like terms together. [ie. the constants]	$y = 4\left(x - \frac{3}{8}\right)^2 + \frac{7}{16}$
7. Statement – min/max value.	Minimum of $\frac{7}{16}$ at $x = \frac{3}{8}$.

EXERCISE: Determine the minimum or maximum value of each quadratic relation using two of the three methods for each equation.

1. $f(x) = x^2 + 5x - 1$

2. $f(x) = -x^2 - 7x$

3. $f(x) = 2x^2 + 6x - 1$

4. $f(x) = -5x^2 + 10x + 5$

5. $f(x) = \frac{1}{2}x^2 - 2x + 1$

6. $f(x) = \frac{2}{3}x^2 - 2x + 5$

7. $f(x) = -6x^2 - 2x + 2$

8. $f(x) = 10x^2 - 4.2x + 1.8$

9. $f(x) = 3 - 2x + 3x^2$

10. $f(x) = -0.06x^2 - 6x + 1$

ANSWERS: In no particular order,

Max of 49/4 at x=-7/2
Min of -29/4 at x=-5/2
Max of 13/6 at x=1/3
Max of 151 at x=-50
Min of 7/2 at x=3/2

Min of -11/2 at x=-3/2
Min of 8/3 at x=1/3
Min of 1.359 at x=0.21
Min of -1 at x=2
Max of 10 at x=1