MDM 4U
PROBABILITY DISTRIBUTIONS

Many probability experiments have numerical outcomes—outcomes that can be counted or measured. A ***random variable, X***, has a single value (denoted x) for each outcome in an experiment. For example, if X is the number rolled with a standard die, then x has a different value for each of the six possible outcomes, namely 1 through 6. Random variables can be discrete or continuous. A **DISCRETE** random variable is a random variable that has countable values; for instance, the sum of two dice has only integer values 2 through 12. **CONTINUOUS** variables have an infinite number of possible values in a continuous interval. The focus of this chapter in on distributions involving discrete random variables.

A discrete **PROBABILITY DISTRIBUTION** describes the **probability** of occurrence of each value of a **discrete** random variable. When calculating probability distributions, the random variable X, that is to be measured, is defined first. The probability of a random variable having a particular value x is represented as **P(X=x)**. A probability distribution typically shows the probabilities of all possible outcomes of an experiment. The sum of the probabilities in any distribution is 1.

EXPECTATION, or the predicted ***average*** of all possible outcomes of a probability experiment, is

$$E(x) = \sum_{i=1}^{n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \cdots + x_n P(x_n)$$

EXAMPLES:

① A) Determine the probability distribution for the sum rolled with two dice.

Let X = random variable
= sum of 2 dice

B) Determine the expected sum of the two dice.

X=sum	P(X=x)	xP(x)
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
Σ		

② A game consists of rolling a die. If an odd number is rolled, you lose points equal to the face value. If an even number is rolled, you gain points equal to the face value.

Let X = random variable
= points on each roll

A) Determine the probability distribution for the points won in the game.

X=points	P(X=x)	xP(x)
-1		
Σ		

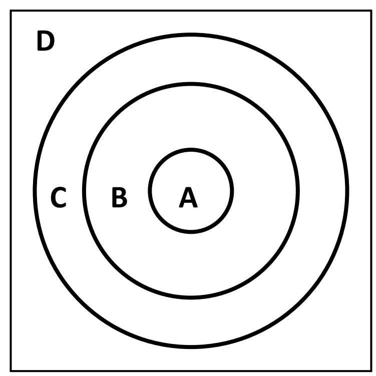
B) Determine the expected number of points on each roll. Is it a fair game?

③ In a lottery, there are 10 000 tickets to be sold at $100 each to raise funds for cancer research. If the prizes are 1-$50 000 prize, 10-$5 000 prizes, 50-$1 000 prizes, and 100-$100 prizes, [assuming all tickets are sold]

A) determine the probability of winning a prize.
B) illustrate the probability distribution for the amount of money won.
C) determine the expected profit per ticket.
D) What should the lottery prize organization charge for each ticket in order to make a 30% profit?

④ A dartboard (35 x 35 cm) has circles with radii 6 cm, 10 cm, and 16 cm. A player earns 25 points if a dart lands in A; 15 points if lands in B; and 5 points if it lands in C, but loses 25 points if it lands outside of the rings.
[Assume all darts hit the board.]

A) Determine the probability distribution for the number of points won.
B) What is the expected number of points for each dart thrown?



⑤ A slot machine at a casino has 3 rollers, each containing the symbols indicated in the chart along with their corresponding probabilities of stopping on the payout line.

cherries	$ signs	stars	apples	sevens
25%	20%	10%	5%	2%

A) Determine the probability distribution of the amount of money won given the following payout amounts: $2 for 3 cherries in a row, $10 for 3 dollar signs in a row, $100 for 3 stars, $500 for 3 apples, and $1000 for 3 sevens.
B) If it costs $2 to play each time, what is the expected amount of money won? What would the casino's profits be on 1000 trials?