MHF 4U
PROPERTIES of GRAPHS of FUNCTIONS

The graphs of functions can be distinguished not only by their shapes, but also by their properties. The properties or characteristics include, but are not limited to:

① **Domain and Range** – **Interval Notation**

Eg. $\{x \in R | x > 2\} \rightarrow (2, \infty)$ $\{x \in R | -3 < x \leq 2\} \rightarrow (-3, 2]$

$\{x \in R | x \leq -1\} \rightarrow (-\infty, -1]$ $\{x \in R | x \geq 5\} \rightarrow$_____

$\{x \in R | 0 \leq x \leq 10\} \rightarrow$_____ $\{x \in R | x < 4\} \rightarrow$_____

$\{x \in R | x \neq -3\} \rightarrow$_____ $\{x \in R | x \neq 0, 6\} \rightarrow$_____

RULES:

② **X-intercepts (or zeros) and Y-intercept**

③ **Vertical and Horizontal Asymptotes**

④ **Continuity and Discontinuity**

- Vertical asymptotes
- Holes in the line/curve

⑤ **Intervals of Increase and Decrease**

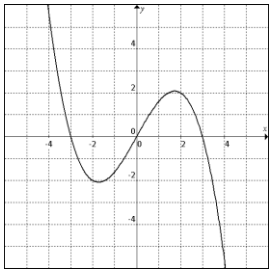
⑥ **Maximums and Minimums** -- coordinates of turning points of a graph

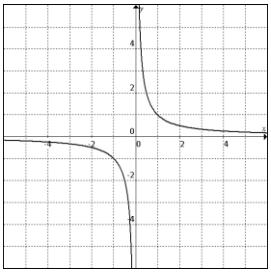
⑦ **End Behaviour** = how do the y −coordinate behave as x goes to the far left and as x goes to the far right of the graph?

Symbols: $as\ x \to -\infty, y \to ?$

$as\ x \to \infty, \quad y \to ?$

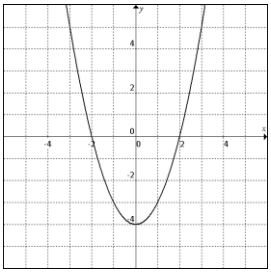
Eg.

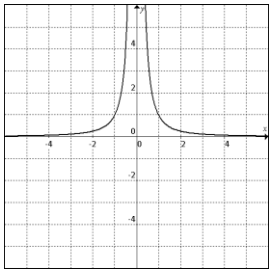


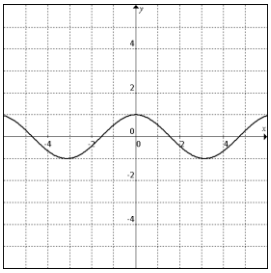


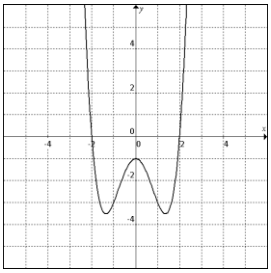
⑧ **Symmetry (even/odd)** – [examine each group of graphs and describe the Rules below]

EVEN SYMMETRY:

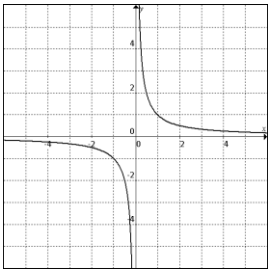


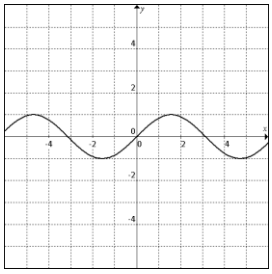


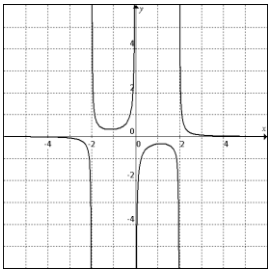


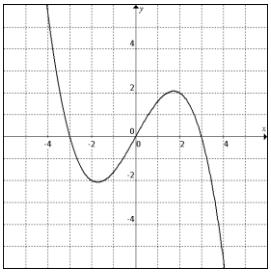


ODD SYMMETRY:









RULES:

- Given the graph of a function, we can describe its characteristics.
- Given the characteristics of the graph of a function, we can sketch the function and state the equation of the function.

EXERCISE:

1. Describe the distinguishing characteristics between each pair of functions.

   A) $f(x) = x^2;\quad g(x) = \sqrt{x}$

   B) $f(x) = \sin x\,;\ g(x) = x^2$

   C) $f(x) = 2^x;\quad g(x) = x^2$

   D) $f(x) = |x|;\quad g(x) = x^2$

2. Consider $f(x) = 4 - x^2$. Describe the characteristics of f(x).

3. Sketch the function with the following characteristics. Also, provide a possible equation for the function:

   A) D = (-∞, ∞), R = (-∞,4), decreasing on (-∞,∞).

   B) As $x \to -\infty$, $y \to 1$, and as $x \to \infty$, $y \to 1$, increasing on (-∞,0) U (0, ∞).

4. Determine algebraically whether each function has even, odd, or neither symmetry.

   A) $f(x) = x^2 - 4$

   B) $g(x) = \frac{4x^2}{x-2}$