SCH 3U

PART 1: SCIENTIFIC NOTATION

		EXAMPLE ①:	EXAMPLE ②:
RULE #1:	Move the decimal point so that there is only one non-zero digit to the left of the decimal point.	362.45	0.000 810
RULE #2A:	Determine what power of 10 will be used. If the original number is larger than 10, then the exponent is positive and equal to the number of places the decimal point moved in Rule #1.		
RULE #2B:	If the original number is less than 1, then the exponent is negative and equal to the number of places the decimal point moved in Rule #1.		

Convert to scientific notation.

1. 82.0
2. 246.02
3. 0.005 01
4. 0.013 407 6
5. 0.000 030
6. 0.140 0
7. 87.000
8. 87 000
9. 8 700.0
10. 2 060
11. 0.000 009 000
12. 2.999

Convert to standard notation.

1. 1.101×10^{2}
2. 9.5×10^{-5}
3. 7.03×10^{4}
4. 6×10^{-6}
5. 5.0×10^{5}
6. 1×10^{-4}
7. $4.000\ 010 \times 10^{3}$
8. $4.000\ 010 \times 10^{-1}$
9. 8.050×10^{-8}
10. 6078×10^{-6}
11. 0.000390×10^{-2}
12. $0.008\ 000 \times 10^{2}$

PART 2: SIGNIFICANT DIGITS

RULE #1: Starting from the left, mark the first non-zero.

RULE #2A: If NO digits follow the decimal point, mark the last non-zero.

RULE #2B: If digits follow the decimal point, mark the last digit (even if it is a zero).

RULE #3: The number of significant digits will equal the count of digits from the first significant digit marked to the last significant digit marked.

Determine the number of significant digits for each.

1.	80.9	2.	246.02	3.	0.005 01
4.	0.013 407 6	5.	0.000 30	6.	0.140 0
7.	87.000	8.	87 000	9.	8 700.0
10.	2 060	11.	0.000 090	12.	2.999
13.	4.0×10^{-5}	14.	5.640×10^{3}	15.	1.001×10^{-8}
16.	9000.	17.	9.00×10^{3}	18.	9.000×10^{3}
19.	9×10^{3}	20.	9.0000×10^{3}	21.	9.0×10^{3}

PART 3: ROUNDING NUMBERS

When rounding, examine the digit following the digit to be rounded. The digit to be examined is the first digit to be dropped. If the first digit to be dropped is...

① **LESS than 5... drop it and all digits to the right; keep digit to be rounded the same.**

EXAMPLES:

Round 62.53<u>4</u>7 to four significant digits. Look at the fifth digit (underlined digit). Since the 4 is less than 5, drop every digit after the 4 significant digits. 62.53<u>4</u>7 rounds to 62.53.

Round 43 <u>3</u>68 to two significant digits. Look at the third digit; since the 3 is less than 5, drop every digit after the first 2 significant digits. To maintain place value, the digits being dropped are replaced with zeros. 43 <u>3</u>68 becomes 43 000.

② **MORE than 5... drop it and all digits to the right; increase digit to be rounded by 1.**

EXAMPLES:

Round 3.78<u>7</u>21 to three significant digits. Look at fourth digit. Since 7 is greater than 5, round 3.78<u>7</u>21 up to 3.79.

Round 24.8<u>5</u>14 to three significant digits. Look at fourth digit. Since the value following the 8 is more than 5, 24.8<u>5</u>14 becomes 24.9.

③ **ONLY 5... round the digit that is to be rounded off so that it will be even. Keep in mind that zero is considered to be even when rounding off.**

EXAMPLES:

Round 726.83<u>5</u> to five significant digits. Look at sixth digit. Since it is just a 5, also consider the fifth digit. Since the fifth digit is a 3 (odd number), round 726.83<u>5</u> to 726.84.

Round 48.5 to the nearest units. Look at tenths place. Since it is just a 5, consider the digit to be rounded. Since it is an 8 (even number), 48.5 is rounded to 48.

<u>NOTE:</u> Place value must be retained, so any dropped digits that are ***left*** of a decimal point are replaced with zeros.

EXAMPLES:

Round each number as indicated:

1. 23.95 to one significant digit.

2. 84 500.6 to two significant digits.

3. 84 500 to i) 2 sig dig ii) 1 sig dig iii) 3 sig dig

4. 989.95 to i) 1 sig dig ii) 2 sig dig iii) 3 sig dig iv) 4 sig dig

5. 3555 to i) 1 sig dig ii) 2 sig dig iii) 3 sig dig iv) 4 sig dig

6. 895.65 to i) 1 sig dig ii) 2 sig dig iii) 3 sig dig iv) 4 sig dig

7. 6.5950×10^5 i) 1 sig dig ii) 2 sig dig iii) 3 sig dig

   [when rounding numbers in scientific notation, ignore the power]

PART 4: EXACT NUMBER VS. MEASUREMENTS

Measurements always have a certain number of significant digits due to the precision of the instrument used.

Exact numbers have no uncertainty because they are counting numbers, such as I have 3 books or I have a dozen daffodils. Since there is no uncertainty, we can assume that the number actually has an infinite number of significant digits. This would not figure in to our rounding while doing calculations.

3 books = 3.0000000000000000000000... books

Eg.	Total mass	=	(mass per book) x (number of books)	
		=	3.64 g x 3	(3 significant digits x infinite sig dig)
		=	10.92 g	
		=	10.9 g	(round off to 3 sig dig)

Eg. 17 cm is a measurement, therefore 2 sig dig.
17 books is an exact number, therefore infinite sig dig.

Determine the number of significant digits.

1. 35 grams	2. 28	3. 10 houses
4. 4.85 km	5. 9 baseballs	6. 0.040 750 sec
7. 9.53 mL salt water	8. 9.5 kg	9. 20 questions

PART 5: SIGNIFICANT DIGITS IN CALCULATIONS

When multiplying and dividing, the answer will have the same number of significant digits as the number in the question with the ***fewest significant digits***.

Eg. 4.632 x 0.90

Eg. 0.750 ÷ 1.234

When adding and subtracting, the answer will have the same number of decimal places as the number in the question with ***fewest number decimal places***.

Eg. 4.632 + 0.90

Eg. 5.750 – 1.4 + 13.60

PART 6: SIGNIFICANT DIGITS, CALCULATIONS & MEASURING ERROR

All measurements tend to have error associated with them. The degree of error depends on the device used and, to a smaller extent, the level of confidence of the user.

All digits quoted in a measurement consist of significant digits; the last digit is always the uncertain digit, which means that all other digits quoted by a technician are certain.

Digital devices typically have an error of ± 1 of the last measured digit (the uncertain digit). Analog devices (such as a ruler, glassware for measuring volumes of liquids, a manual weigh scale, etc) have an error of $\pm 2\ to\ 5$ of the uncertain digit. ***The smaller the unit of measure, the greater the error.***

EXAMPLES OF ERROR: For a digital device, a pencil may weigh $23.03\ \pm 0.01\ grams$

For a manual device, the pencil may weigh $23.1\ \pm 0.2\ grams$

When using measured values (which include error) in a calculation, the error associated with the calculation compounds according to the type of calculation.

There are a variety of methods for calculating error involving measured values.
One method (illustrated here) is to perform the following steps:

① perform actual calculation, without regard to any error.

② perform the calculation again, using either HIGHS or LOWS of the measured values.

③ error = ② − ①

④ Final quoted = [actual calculation] $\pm$ [error calculated in ③]

ERROR is always quoted using 1 significant figure and the FINAL CALCULATION using measurements involving error is always quoted using the same place value as the error.
See example below:

EXAMPLE: A rectangle has a length measured at $15.86 \pm 0.02\ cm$ and a width measured at $9.63 \pm 0.02\ cm$. Determine the perimeter and area of the rectangle.

STEP ①

ACTUAL PERIMETER:

$P = 2L + 2W$

$P_{actual} = 2(15.86) + 2(9.63)$
$P_{actual} = 50.98\ cm$

ACTUAL AREA:

$A = LW$

$A_{actual} = (15.86)(9.63)$
$A_{actual} = 152.7318\ cm^2$

STEP ②

Using $\boldsymbol{L_{high} = 15.86 + 0.02 = 15.88}$

$\boldsymbol{W_{high} = 9.63 + 0.02 = 9.65}$

$P_{high} = 2(15.88) + 2(9.65)$
$P_{high} = 51.06\ cm$

$A_{high} = (15.88)(9.65)$
$A_{high} = 153.242\ cm^2$

STEP ③

$ERROR = P_{high} - P_{actual}$
$= 51.06 - 50.98$
$= 0.08$

$ERROR = A_{high} - A_{actual}$
$= 153.242 - 152.7318$
$= 0.5102$

STEP ④

$FINAL\ P = 50.98 \pm 0.08\ cm$

$FINAL\ A = 152.7 \pm 0.5\ cm^2$