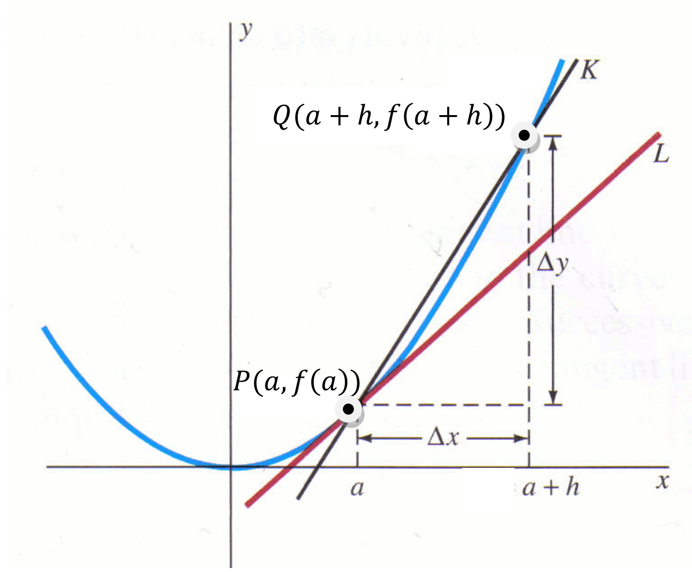
MCV 4U

USING LIMITS to find SLOPES of TANGENTS

Consider the diagram to the right. To find the tangent to the curve at point $P(a, f(a))$:



1. Consider a point nearby.
   $Q(a+h, f(a+h))$

2. Compute the slope of the secant PQ.

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{h}$$

3. Let *Q* approach *P* along the curve by letting Δx *or* h approach 0.

4. The slope of the secant will approach the exact value of the slope of the tangent, $\boldsymbol{L}$, at point *P*, more and more closely as h approaches 0.

5. This process of calculating a quantity by estimating, and then making the estimate more and more accurate until it is exact, is known as **determining a limit**.

6. Therefore, using Notation of Limits:

$$m_{tan} = \lim_{h \to 0} m_{PQ}$$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

DISCONTINUITY & the SLOPE of a TANGENT

The slope of the tangent to a curve is determined using $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

The expression $\frac{f(a+h) - f(a)}{h}$ contains a hole at $h = 0$. So, to find the limit as h approaches zero, the hole is first removed by applying algebraic steps to cancel h from the denominator, similar to finding the limit of any function close to a removable discontinuity.

EXAMPLES: Find the ***slope*** of the tangent to each curve at the given value of x.
Then determine the ***equation*** of the tangent line in the form $y = mx + b$.

① $f(x) = -x^2 - 3x - 3;\quad x = 1$

$m_{sec} = \frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{h}$ [where "a" represents given x value; sub in at end of limit]

$= \frac{[-(a+h)^2-3(a+h)-3]-[-a^2-3a-3]}{h}$

$= \frac{[-a^2-2ah-h^2-3a-3h-3]+a^2+3a+3}{h}$

$= \frac{-2ah-h^2-3h}{h}$

$= -2a - h - 3$

$m_{tan} = \lim_{h\to 0} (-2a - h - 3)$

$= -2a - 3$

At $x = 1$, the slope of the tangent...
$m_{tan} = -2(1) - 3$
$= -5$

And the equation of the tangent is...
$y = -5(x - 1) - 7$
$y = -5x - 2$

② $f(x) = \frac{3x}{x-2};\quad x = 4$ and $x = 2$

③ $f(x) = -2\sqrt{x+1};\quad x = 3$ and $x = -1$

④ $f(x) = \frac{2}{\sqrt{x-3}};\quad x = 7$

⑤ $f(x) = \sqrt{x^2+1};\quad x = -2$ and $x = 0$

Answers (in no particular order and may be used more than once):
0, -1/6, undefined, -2, -2/√5, -1/8, -1/2, -5, -3/2