MCR 3U

PART 1: TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS

Parent form of an exponential function → $y = A^x$, where A>0 and A≠1.

The family of exponential functions undergo transformations similar to those found with other functions, such as $y = x^2$, $y = x^3$, $y = \sqrt{x}$, and $y = |x|$.

Compare the first 2 columns of the chart below. Describe the transformation that is applied to the parent function.

Parent	Transformed Function	Transformation?
$y = 2^x$	$f(x) = 3(2)^{x-1}$	
$y = 10^x$	$n(x) = -10^x$	
$y = 3^x$	$g(x) = 9^x - 3$	
$y = 1.03^n$	$A(n) = 100(1.03)^n$	
$y = \left(\frac{1}{2}\right)^x$	$h(x) = \left(\frac{1}{2}\right)^x + 2$	

The general form of a transformed exponential function is...

$$y = A^x \longrightarrow f(x) = a(A)^{k(x-\mathrm{h})} + v$$

parent → transformed function

where
A = ***base*** of the parent exponential function
a = vertical expansion/compression/reflection in x −axis
v = vertical shift
k = horizontal expansion/compression/reflection in y −axis
h = horizontal shift

EXERCISE:

① For each exponential function,

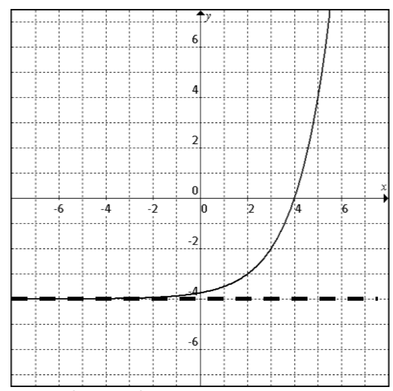
- Use exponent rules to simplify the equation.
- Highlight the parent graph and state its equation
- Describe the transformation to the parent function and write the mapping.
- State the location of the horizontal asymptote.
- Complete a table using $x = -1, 0, 1$. Sketch the transformed function.
- Describe domain and range.

<u>**EXAMPLE:**</u> $f(x) = 8(2)^{x-5} - 4$

- $f(x) = 2^3(2)^{x-5} - 4$
  $f(x) = (2)^{x-2} - 4$

- PARENT: $y = 2^x$
- TRANSFORMATION: shift right 2 and down 4
- Horizontal Asymptote: $y = -4$
- TABLE:

x	$y = 2^x$	$(x+2, y-4)$
-1	0.5	(1, -3.5)
0	1	(2, -3)
1	2	(3, -2)



$D = \{x \in R\}$; $R = \{y \in R, y > -4\}$

A) $f(x) = 4(2^x) - 2$ B) $f(x) = 9(3^{x+3})$ C) $f(x) = \frac{1}{8}\left(\frac{1}{2}\right)^x + 1$

D) $f(x) = \frac{1}{8}(2)^{x+1}$ E) $y = -3(3)^{x+2} + 1$ F) $f(x) = \frac{1}{16}(2)^{-x-3}$

G) $y = 3\left(\frac{1}{2}\right)^{x-3} + 3$ H) $y = 2\left(\frac{1}{5}\right)^{x-2} - 25$ I) $f(x) = -2\left(\frac{1}{3}\right)^x - 2$

J) $y = 3(2)^x - 3$ K) $y = -(3)^{x+1} - 2$ L) $y = -\frac{1}{2}\left(\frac{1}{2}\right)^{x+2} + 2$

② For each function in #1 A-F, evaluate $f(-1)$, $f(0)$, $f(2)$, and $f(x) = 0$.

PART 2: DETERMINING the EQUATION of the EXPONENTIAL FUNCTION

Consider any PARENT Exponential Function:

2 key points of the parent include → **(0, 1) & (1, A)**

If we locate the images of these 2 parent points, we can determine the equation in the form $f(x) = a(A)^{(x-h)} + v$.

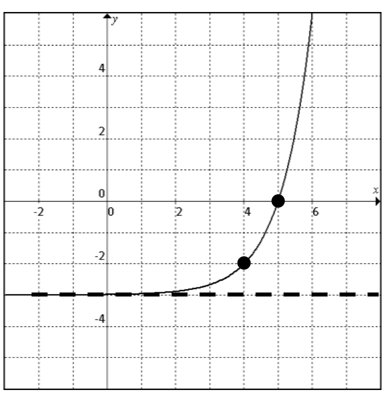
STEP ① Sketch the horizontal asymptote → value of v.

STEP ② Locate 2 points (with integer coordinates) nearest the HA.

- Point closest to HA provides horizontal shift (h) from (0, 1) AND any vertical expansion (a) counting from HA.
- 2nd point determines base (A) – count from HA to this point, however, we also need to account for any vertical expansion.
- INCREASING function → "A" = whole number.
  DECREASING function → "A" = reciprocal of the whole number.

Determine the equation of each exponential function in the form $y = a(A)^{(x-h)} + v$.

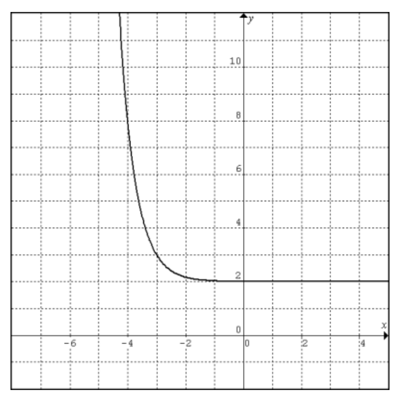
1.



v = -3
h = right 4
a = 1
A = 3 [increases]

$f(x) = 3^{x-4} - 3$

2.



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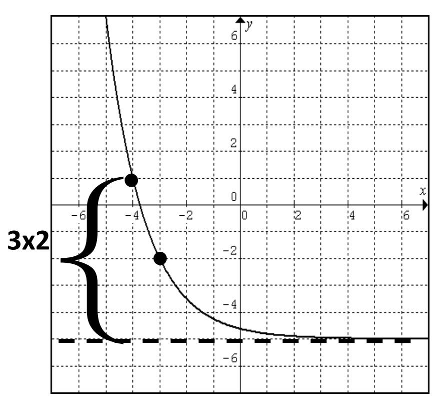
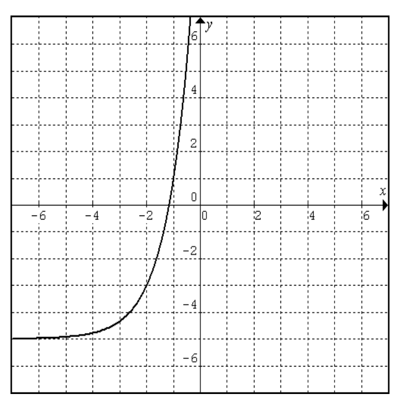


v = -5
h = left 3
a = 3
A = ½ [decreases]

$f(x) = 3\left(\frac{1}{2}\right)^{x+3} - 5$

4.



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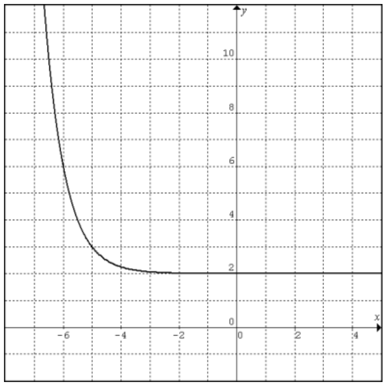

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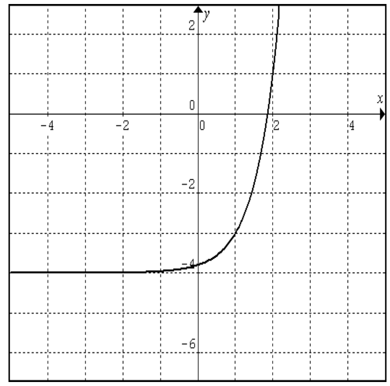

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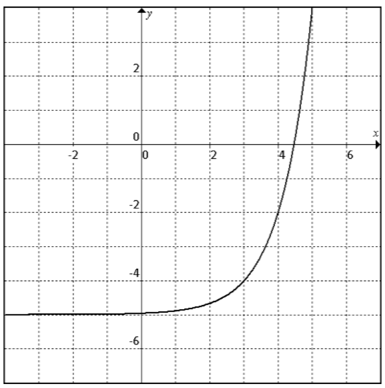

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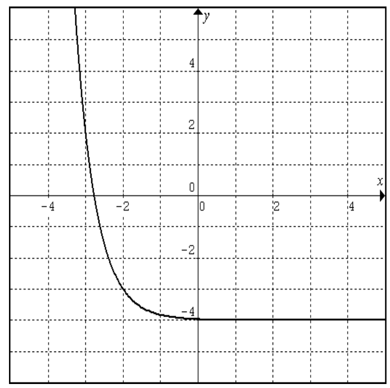

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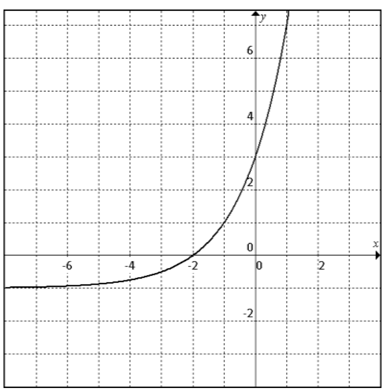

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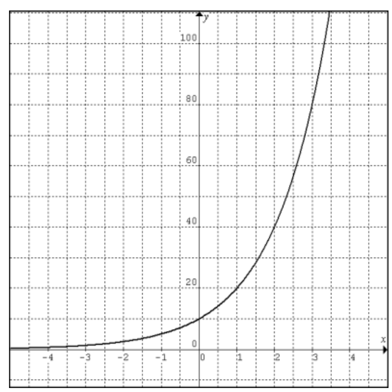
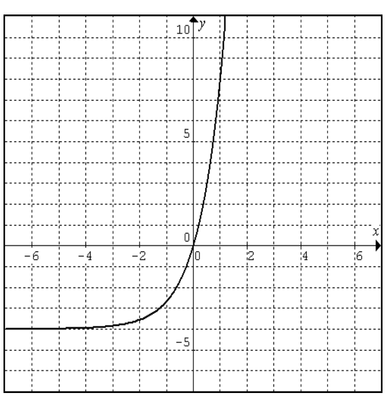


Express in the form $y = a(A)^x$.

11.



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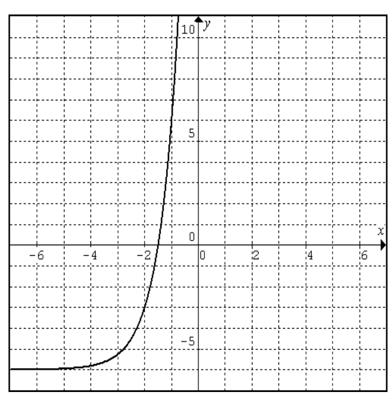

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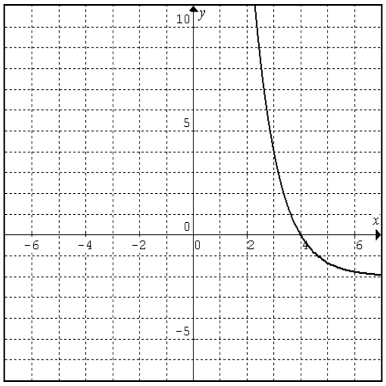

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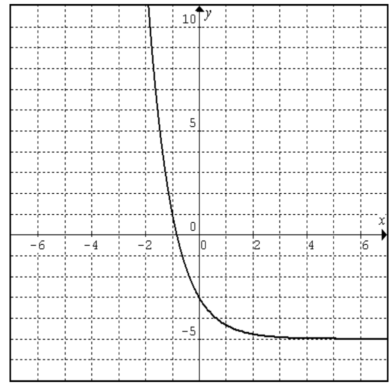

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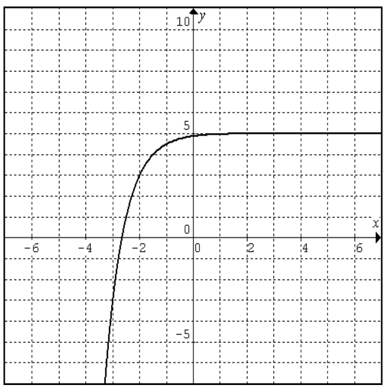

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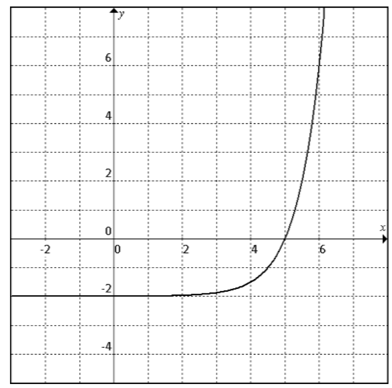

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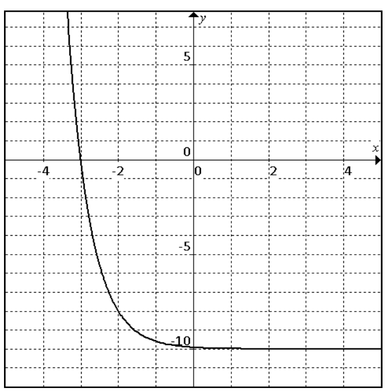

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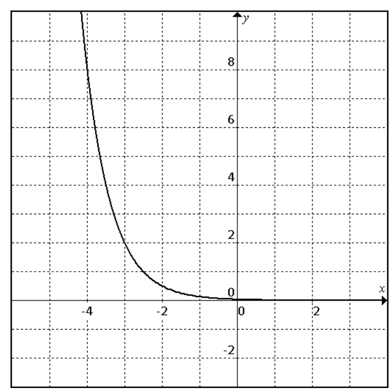

ANSWERS: (in no particular order)

Y=3^(x-4)-3	Y=(1/6)^(x+3)+2	Y=4(3)^x-4	Y=2(1/4)^(x+3)	Y=3^(x-3)-5
Y=-2(1/4)^(x+2)+5	Y=2^(x+2)-1	Y=2(1/5)^(x+2)-10	Y=10(2)^x	Y=2(3)^(x+2)-5
Y=3(1/2)^(x+3)-5	Y=(1/4)^(x+5)+2	Y=2(1/3)^(x-4)-2	Y=-8^(x-3)+5	Y=3(4)^x-6
Y=2(1/3)^x-5	Y=(1/6)^(x+2)-4	Y=2(4)^(x-5)-2	Y=5^(x-1)-4	Y=3(4)^(x+2)-6