MCR 3U

PART 1: TRANSFORMATIONS OF $y = \sin x$ AND $y = \cos x$

Transformations of functions such as $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = x^3$, and $f(x) = |x|$ can be applied to sinusoidal functions as well. **Descriptions** such as reflection in the x −axis, vertical expansion/compression, horizontal expansion/compression, shift left/right, & shift up/down may be used individually or in combination to describe the transformation. **Mappings** may be used to show how the set of points of a parent sinusoidal graph changes to the transformed graph.

$$f(x) = a\sin[k(x - \mathrm{h})] + v$$

Parent → Transformation

$$(x, y) \rightarrow \left(\frac{1}{k}x + \mathrm{h},\ ay + v\right)$$

GRAPHING TRIG FUNCTIONS -- WITHOUT A TABLE OF VALUES

① **HORIZONTAL & VERTICAL SCALES**:

A) **HORIZONTAL:**
- Let 1 period = 12 spaces
- Determine the value of each space

B) **VERTICAL:** Determine **range** of values.
- parent [-1, 1] → transformed range $[-a + v, a + v]$
- label vertical axis with the max, min and equation of axis.

② Apply ***vertical changes*** to the parent trigonometric graph ***within 1 period***.

③ Apply ***horizontal shift*** to the sketch from step ②.

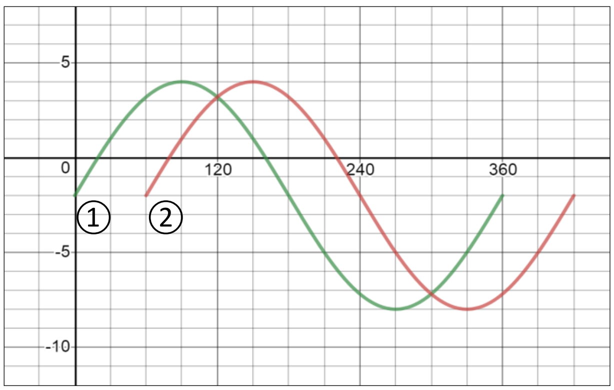
④ Describe the amplitude and period.

EXAMPLE 1: $f(x) = 6\sin[x - 60]^\circ - 2$

Using a mapping & table method:

$(x, y) \rightarrow (x + 60, 6y - 2)$

(x, y)	$(x + 60, 6y - 2)$
(0,0)	(60, -2)
(90,1)	(150, 4)
(180, 0)	(240, -2)
(270, -1)	(330, -8)
(360, 0)	(420, -2)



Amplitude= 6; Period= 360°

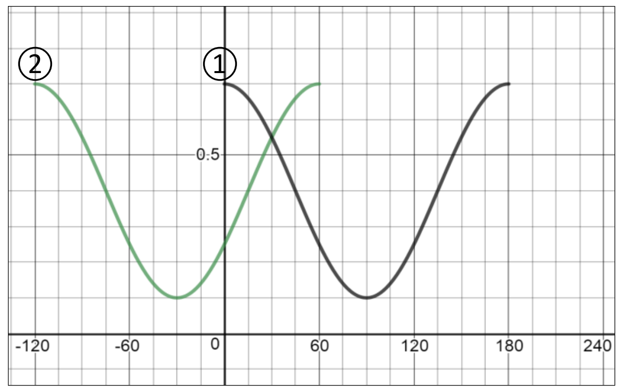
EXAMPLE 2: $f(\theta) = 0.3\cos[2\theta + 240]^\circ + 0.4$

$f(\theta) = 0.3\cos[2(\theta + 120)]^\circ + 0.4$

$(\theta, y) \rightarrow \left(\frac{1}{2}\theta + 120, 0.3y + 0.4\right)$

1 *vertical changes within 180°*

2 *horizontal shift of graph #1*



Amplitude= 0.3; Period= 180°

EXERCISE: Sketch and describe the amplitude and period of each trigonometric function.

1. $\boldsymbol{f(x) = 8\sin(x + 60)^\circ}$

2. $f(x) = \cos(x - 300)^\circ - 4$

3. $f(x) = 5\sin x^\circ - 3$

4. $f(x) = \cos(x + 30)^\circ + 3$

5. $f(n) = 12\sin(n + 180)^\circ - 8$

6. $f(t) = 40\cos(t - 120)^\circ + 10$

7. $P(t) = 0.05\sin t^\circ + 0.02$

8. $\boldsymbol{f(x) = 0.006\cos(x - 75)^\circ - 0.01}$

9. $f(x) = \sin(2x + 60)^\circ$

10. $\boldsymbol{f(x) = \cos\left(\frac{1}{3}x - 20\right)^\circ + 2}$

11. $\boldsymbol{f(x) = 10\sin[3(x + 60)^\circ]}$

12. $f(x) = 4\cos\left(\frac{1}{2}x + 45\right)^\circ - 2$

13. $f(x) = 0.2\sin\left(\frac{1}{2}x - 60\right)^\circ + 0.6$

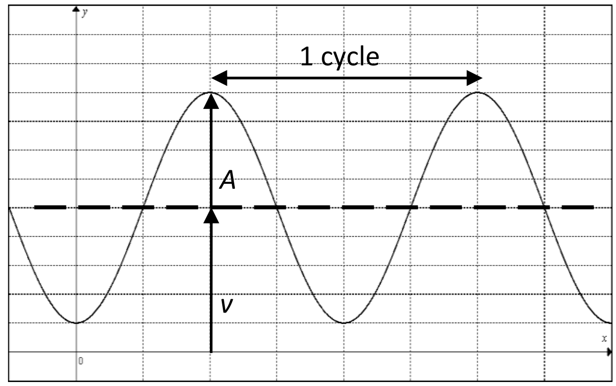
14. $f(x) = 80\cos[2(x + 60)]^\circ - 20$

15. $f(x) = 0.05\sin(3x)^\circ - 0.1$

16. $f(x) = \cos(4x + 180)^\circ + 0.5$

PART 2: DETERMINING the EQUATION of a SINUSOIDAL FUNCTION

To determine the equation of a sinusoidal function given its graph, use the following steps:



1. Draw the horizontal axis of symmetry. Choose between the sine or cosine function to model the graph.

2. Determine the horizontal and vertical scale (unit measure).

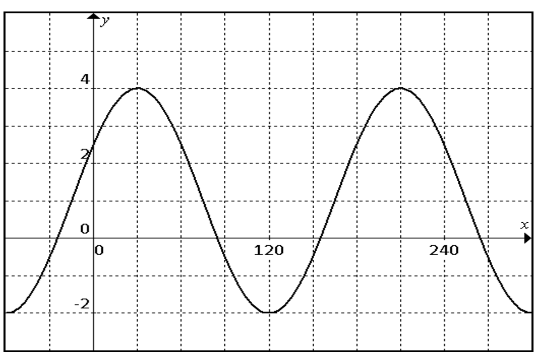
3. Determine the amplitude and vertical shift, relative to the scale of the graph.

4. Measure the length of 1 cycle. Determine the horizontal expansion/compression factor using $k = \frac{360}{1\ cycle}$. Determine the horizontal shift of the starting point of the sin/cos function from the y-axis.

5. Write the model in the form $\boldsymbol{f(x) = Asin[k(x-h)] + v}$ or using cos.

EXAMPLES:

1.



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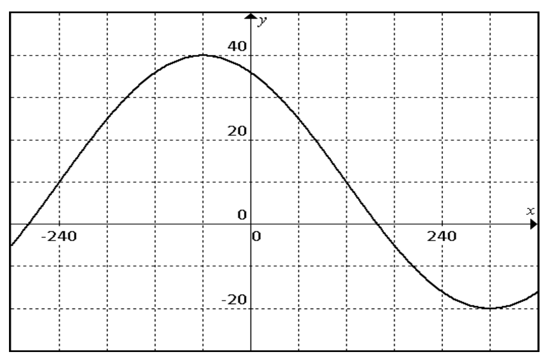

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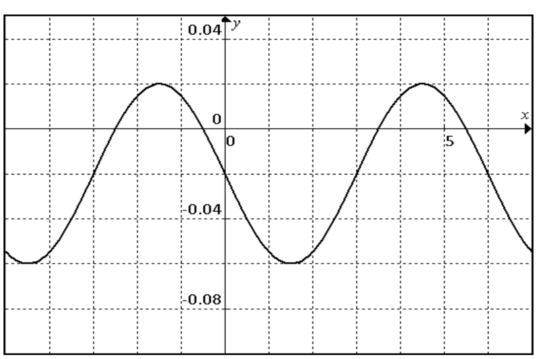

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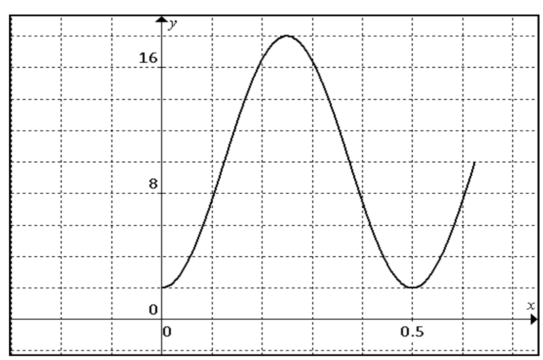

EXERCISE: Determine the equation(s) of each trigonometric function.

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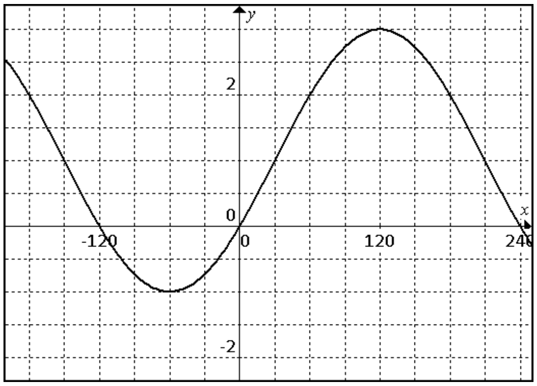

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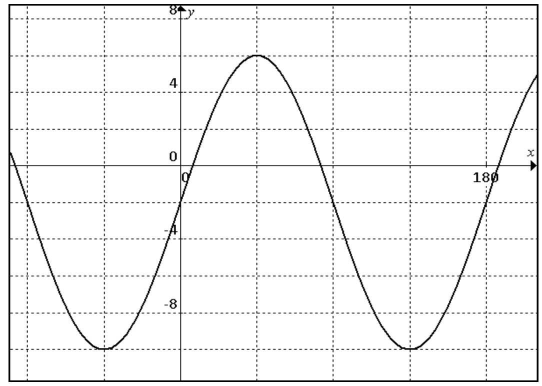

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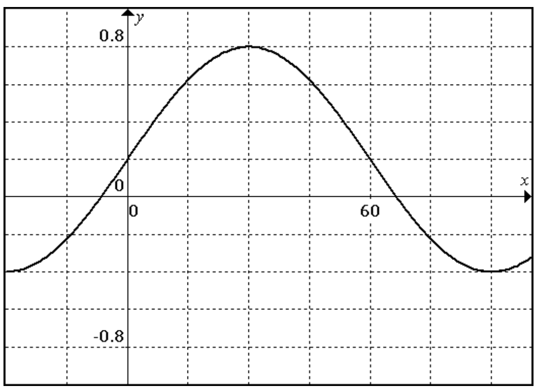

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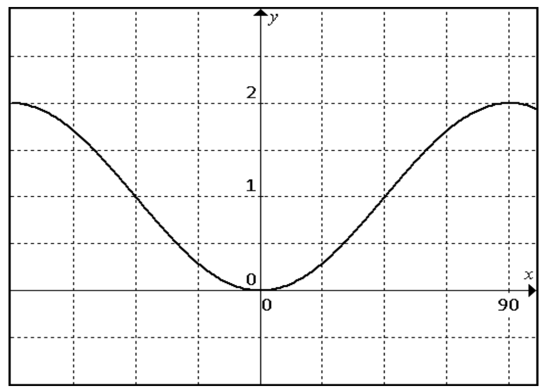

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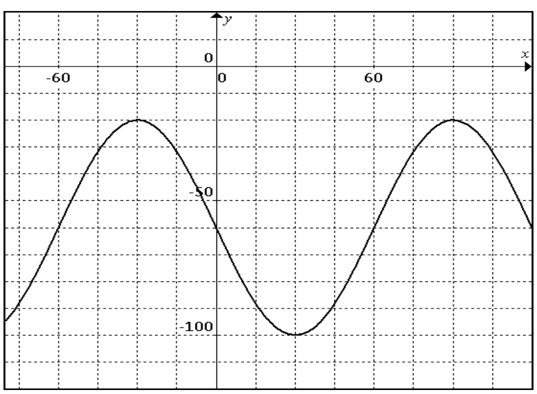

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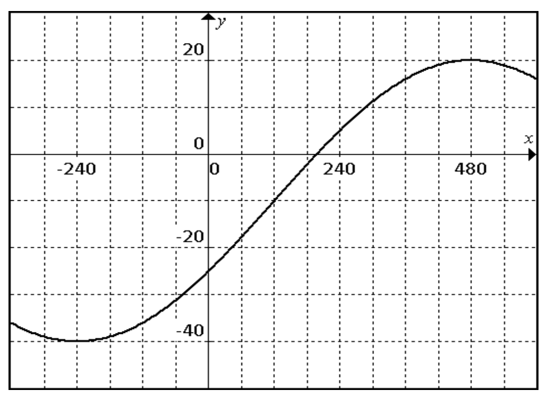

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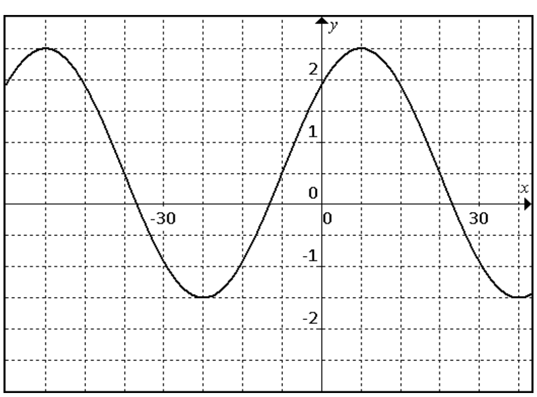

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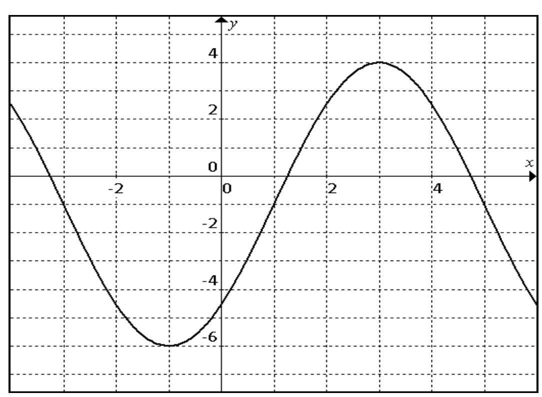

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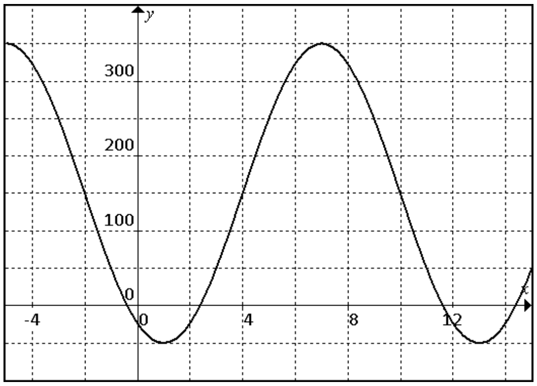

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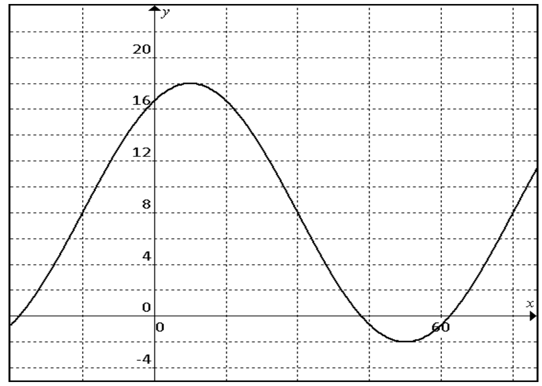

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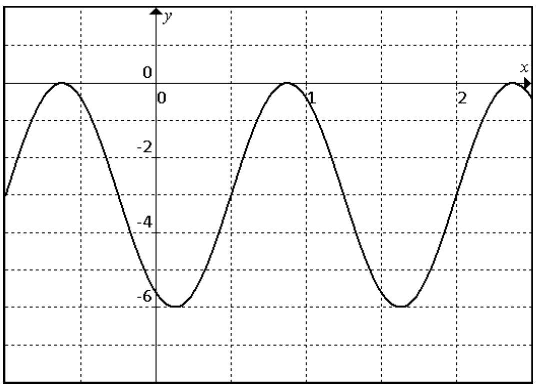

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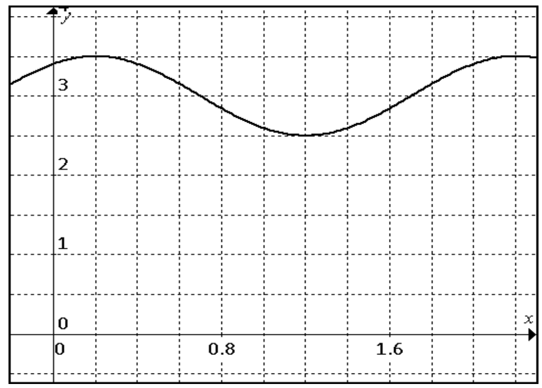

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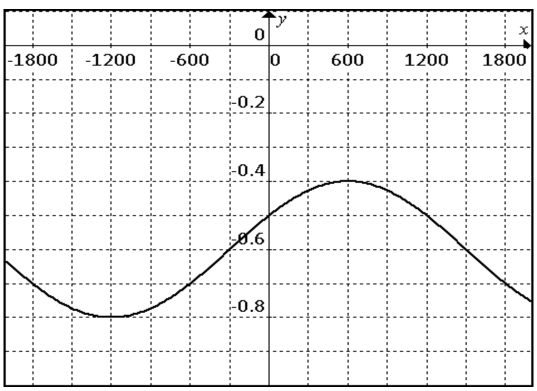

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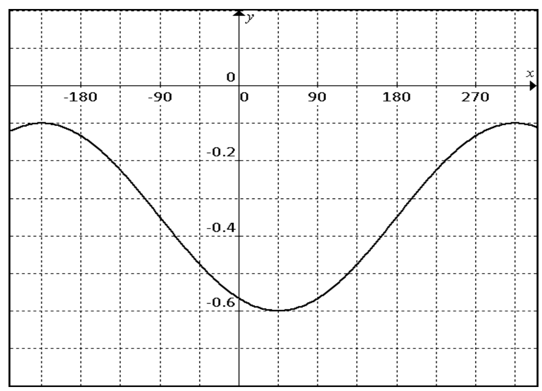

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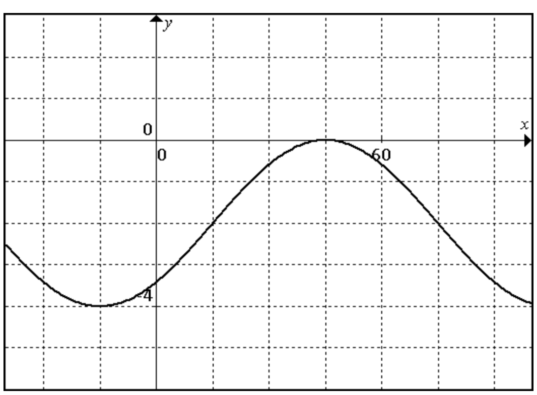

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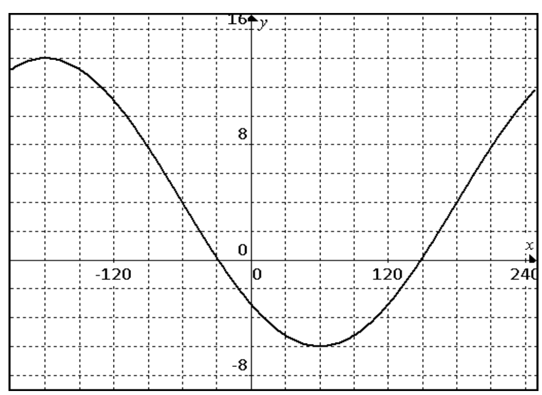

ANSWERS:

y=-10sin[0.75(x-180)]+4;	y=-0.25cos[2/3(x-45)]-0.3;	y=0.5cos[180(x-0.2)]+3;	y=0.6sin3x+0.2;
y=2cos[x-120]+1;	y=-cos2x+1;	y=-200sin[30(x-4)]+150;	y=2cos[3(x-45)]-2;
y=10sin[4(x+15)]+8	y=0.2cos[1/10(x-600)]-0.6;	y=2sin[6(x+30)]+0.5;	y=8sin2x-2;
y=-40sin3x-60;	y=30sin[1/4(x-120)]-10;	y=3sin[240(x-0.5)]-3;	y=5sin[45(x-1)]-1