MCR 3U

PART 1: GRAPHING FUNCTIONS OF THE FORM $y = af[k(x - h)] + v$

By the end of this section, you should be able to apply combinations of transformations to a given relation or function, then describe specific properties of the function, including the domain and range.

The transformations of a function include:

- Expansions/compressions – both horizontally and vertically
- Reflections – both horizontally and vertically
- Translations – shifting a graph left/right, up/down

The ***GENERAL EQUATION*** for a transformed function can be written as...

$$\boldsymbol{y = af[k(x - h)] + v}$$

- $f(x)$ represents the parent function
- $a, k, h,$ and v relate to the transformation

Examples : For each function,

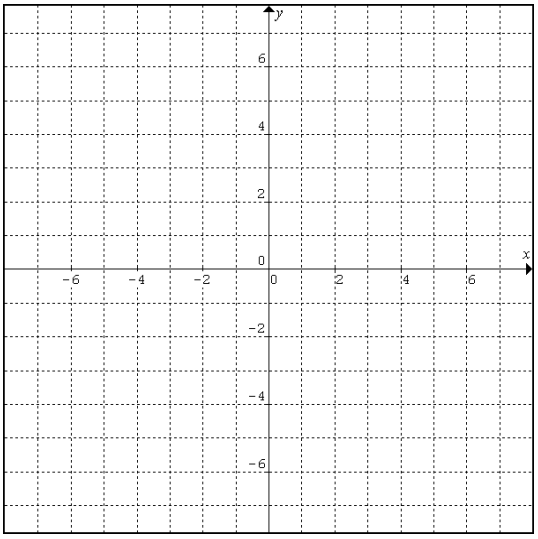
- Describe the transformation (including the mapping statement).
- Prepare a table of values and sketch the function
- State domain/range and intercepts (by algebra)

1. $f(x) = -2\sqrt{2x + 8} + 2$

$f(x) = -2\sqrt{2(x + 4)} + 2$

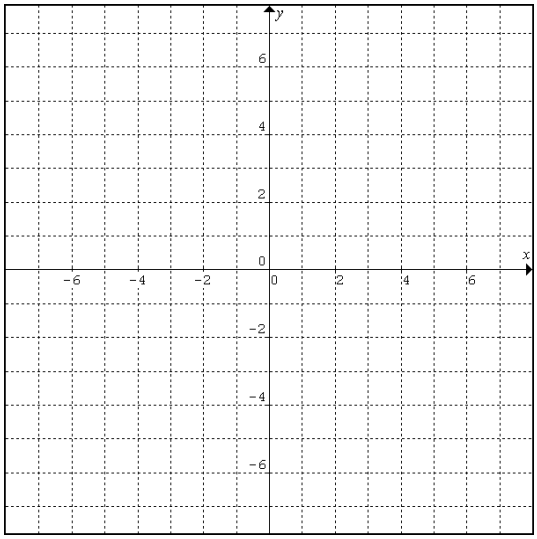
Reflect vertically, expand vertically by 2
Compress horizontally by 1/2
Shift left 4 and up 2
$(x, y) \rightarrow \left(\frac{1}{2}x - 4, -2y + 2\right)$

$y = \sqrt{x}$	$\left(\frac{1}{2}x - 4, -2y + 2\right)$
(0,0)	
(1,1)	
(4,2)	
(9,3)	



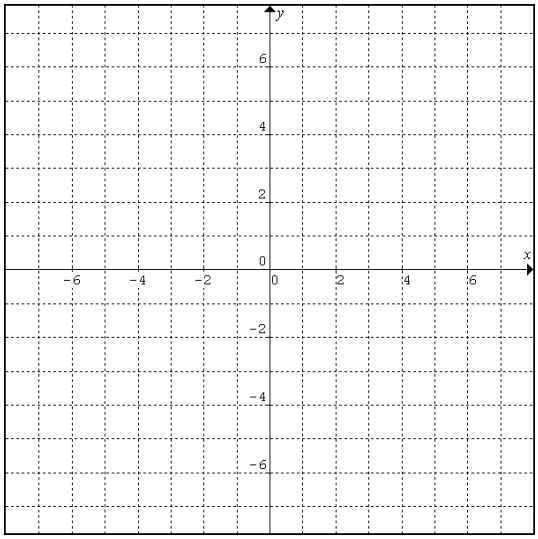
2. $f(x) = \frac{1}{2}\left(\frac{1}{2}x - 2\right)^3 - 4$

$y = x^3$	



3. $f(x) = \left|3 - \frac{1}{2}x\right| - 4$

$y = \lvert x \rvert$	



WORKSHEET

For each function,

- Describe the transformation (and state the mapping).
- Prepare a table of values and sketch the function
- State the domain and range, intercepts (by algebra)

1. $f(x) = \frac{1}{2}\sqrt{x-3} + 2$
2. $f(x) = -2\sqrt{2x+2}$
3. $f(x) = \sqrt{-2x+8} + 2$
4. $f(x) = -\frac{1}{2}\sqrt{2 - \frac{1}{2}x} - 2$
5. $f(x) = -3\sqrt{2-x} - 5$
6. $g(x) = -(x-3)^2 + 1$
7. $g(x) = \frac{1}{2}(x+3)^2$
8. $g(x) = 3(x+1)^2 - 6$
9. $g(x) = -\frac{1}{2}x^2 - 4$
10. $g(x) = -2(x+2)^2 + 8$
11. $a(x) = \frac{1}{3}|-x+4| + 5$
12. $a(x) = -\frac{1}{2}|2x| - 3$
13. $a(x) = 2|-x-1| - 3$
14. $a(x) = -|x-4| + 3$
15. $a(x) = -3|x+2| + 3$
16. $c(x) = \frac{1}{2}(x+2)^3 - 1$
17. $c(x) = -\left(\frac{1}{2}x + 2\right)^3 + 1$
18. $c(x) = 2(-2x-2)^3 - 2$
19. $c(x) = \left(\frac{1}{3}x + 1\right)^3 + 8$
20. $c(x) = -\frac{1}{2}x^3 + 4$

<u>PART 2</u>: DETERMINING the EQUATION of the FUNCTION

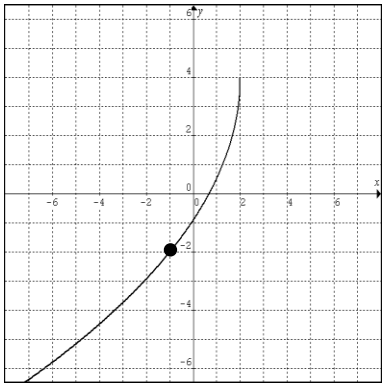
Write the equation of each function in the form $y = aF[k(x-h)] + v$.

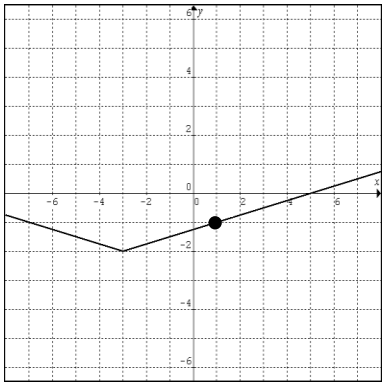
1. $y = \sqrt{x}$ undergoes the mapping defined by $(x, y) \rightarrow \left(-\frac{1}{2}x - 4,\ 5y - 2\right)$.

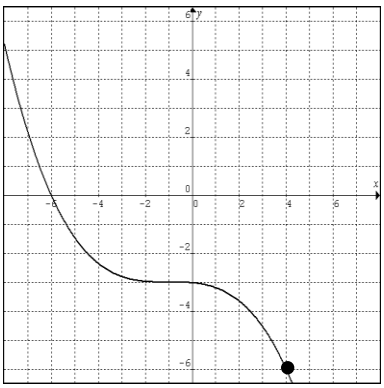
2. $y = \sqrt[4]{x}$ undergoes the mapping defined by $(x, y) \rightarrow \left(-x + 2,\ -\frac{1}{6}y\right)$.

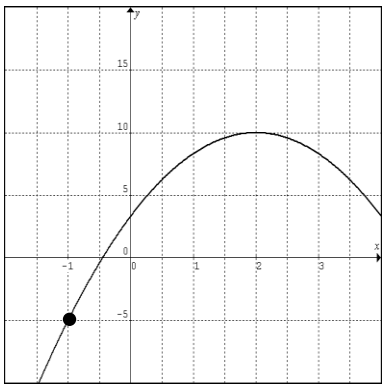
3. $y = x^3$ is reflected horizontally, expanded both vertically and horizontally by 10, shifted right and down 3 units.

4. $y = |x|$ is reflected vertically, compressed horizontally by 1/4, expanded vertically by 5, shifted left 1.

5. 

6. 

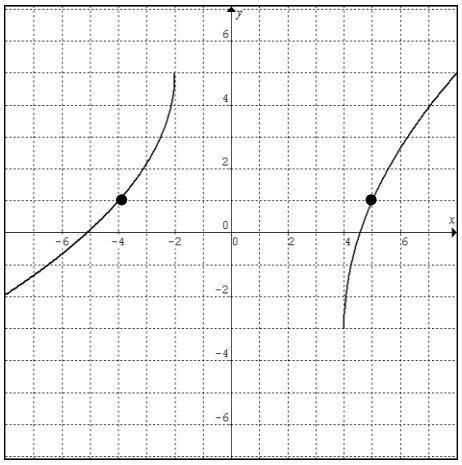
7. 

8. 

EXERCISE:

Determine the equation of each function expressed in the form $\boldsymbol{y = aF[k(x-h)] + v}$

1.



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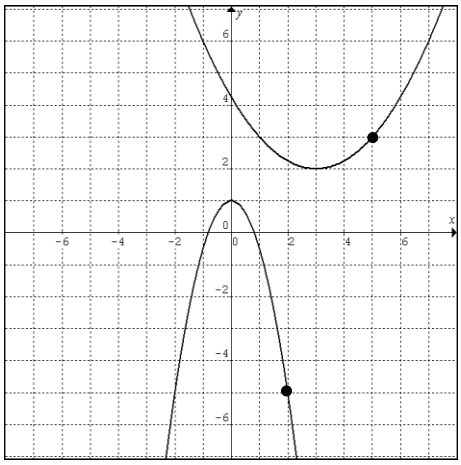

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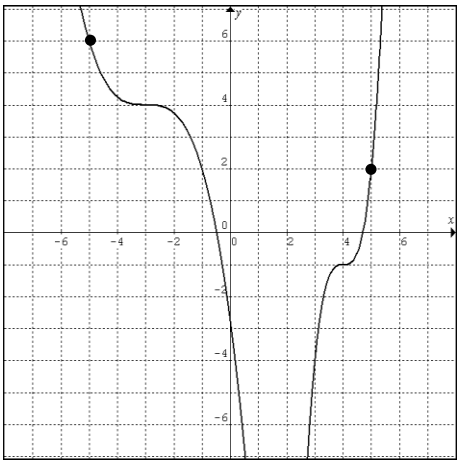

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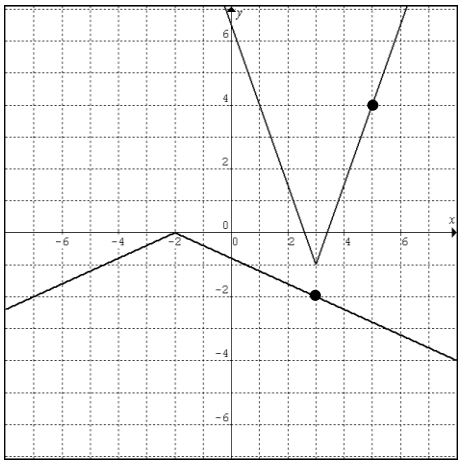

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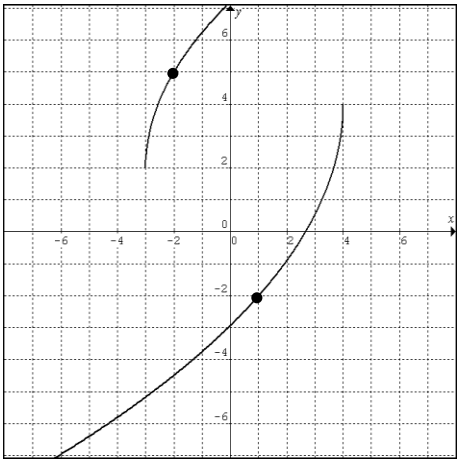

6.