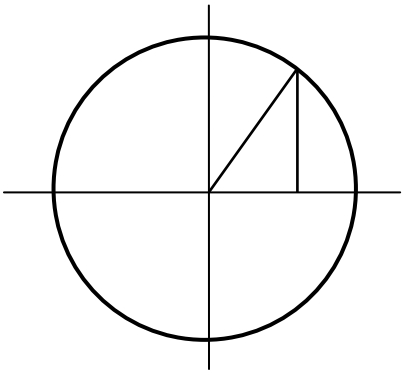
MCR 3U
TRIGONOMETRIC IDENTITIES

The purpose and focus of trig identities are to prove LS = RS of a given equation. ***FORMULAS*** and ***ALGEBRAIC SKILLS*** are used to write trig expressions in other forms, and in many cases, simpler forms. As a result, this section serves the purpose of developing logical and critical thinking skills.

***FAMILIAR FORMULAS*:** ① $csc\theta = \frac{1}{sin\theta}$ ② $sec\theta = \frac{1}{cos\theta}$

***NEW TRIGONOMETRIC FORMULAS*:**

③ Consider the diagram to the right in which a terminal arm has rotated a full 360°. The image of the endpoint of the terminal arm is a circle. If we look at one of the positions of the terminal arm within that circle and label the sides of a right triangle formed with the x −axis, a relationship between trigonometric ratios is formed.



Referring to the right triangle:
horizontal segment = x
vertical segment = y
radius of the circle = r
reference angle = θ

Using the Pythagorean Theorem, $x^2 + y^2 = r^2$.

Dividing each term by r^2, we get $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

which can be rewritten as $\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

and since $cos\theta = \frac{x}{r}$ and $sin\theta = \frac{y}{r}$, $(cos\theta)^2 + (sin\theta)^2 = 1$

which can be simplified to $cos^2\theta + sin^2\theta = 1$ **(3A)**

2 other forms of **(3A)** include $cos^2\theta = 1 - sin^2\theta$ **(3B)**

$sin^2\theta = 1 - cos^2\theta$ **(3C)**

NOTE: *Formula 3A* says $\boldsymbol{cos^2x + sin^2x = 1}$, but $\boldsymbol{cosx + sinx \neq 1}$

④ $tan\theta = \frac{y/r}{x/r} = \frac{y}{x} = \frac{sin\theta}{cos\theta}$

⑤ $cot\theta = \frac{cos\theta}{sin\theta}$

USEFUL ALGEBRAIC SKILLS

- × fractions → $\frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$
- ÷ fractions → $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
- + or − fractions: $\frac{a}{b} + \frac{b}{c} = \frac{ac+b^2}{bc}$
- Factoring
- Expanding
- Substituting using formulas ①-⑤ on front page.

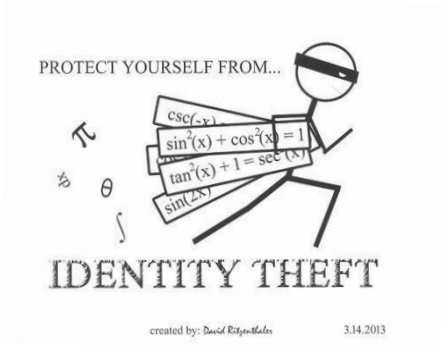
OTHER USEFUL EXPRESSIONS:

1. $sin^2x = (sinx)^2$ 2. $tan^2x = \frac{sin^2x}{cos^2x}$ 3. $sec^2x = \frac{1}{cos^2x}$

4. Rewrite $\frac{\frac{cosx}{sinx}}{\frac{sinx+cosx}{sinx}}$ as $\frac{cosx}{sinx} \div \frac{sinx+cosx}{sinx}$ 5. $sin^2x - sinxcosx = sinx(sinx - cosx)$

EXAMPLES:

1. Simplify the trigonometric expression $\sin x \tan x \cos x$. Evaluate both original and simplified expressions at $x = 25°$ to prove LS = RS.



2. For $tan\theta + cot\theta \equiv sec\theta csc\theta$,

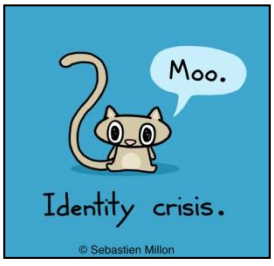
   A) Use $\theta = 60°$ to prove the identity is true.

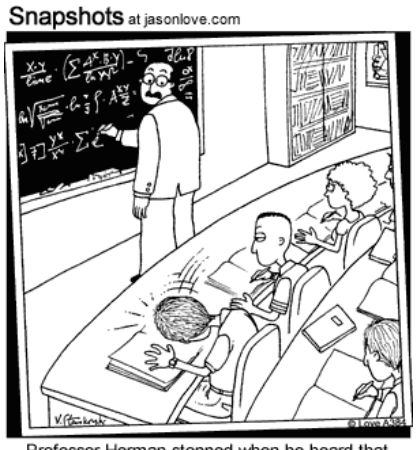
   B) Prove the identity.

3. Prove the identity $(sinx + cosx)^2 \equiv 1 + 2sinxcosx$

4. Prove the identity $\frac{1}{1+cosA} + \frac{1}{1-cosA} \equiv 2csc^2A$

5. Prove or disprove the identity $\frac{1-cot^2B}{cosB+sinB} \equiv cscB - \frac{cotB}{sinB}$





Professor Herman stopped when he heard that unmistakable thud -- another brain had imploded.

EXERCISE: Prove each trigonometric identity.

1. $sinxtanx \equiv \frac{cosx}{cot^2x}$

2. $cscxsecxcotx \equiv csc^2x$

3. $(sinx + cosx)^2 \equiv 1 + 2sinxcosx$

4. $(1 + tanx)^2 \equiv sec^2x + 2tanx$

5. $cosx(secx - cosx) \equiv sin^2x$

6. $sin^3x + sinxcos^2x \equiv \frac{tanx}{secx}$

7. $cotx + tanx \equiv cscxsecx$

8. $cosx + \frac{sin^2x}{cosx} \equiv secx$

9. $1 + \frac{1}{sinx} \equiv (1 + sinx)cscx$

10. $\frac{1}{sinx} - sinx \equiv cotxcosx$

11. $tan^2x(1 + cot^2x) \equiv \frac{1}{1-sin^2x}$

12. $cscx(cosx + sinx) \equiv cotx + 1$

13. $sec^2x - tan^2x \equiv 1$

14. $sinx + \frac{cotx}{secx} \equiv cscx$

15. $1 - cos^4x \equiv 2sin^2x - sin^4x$

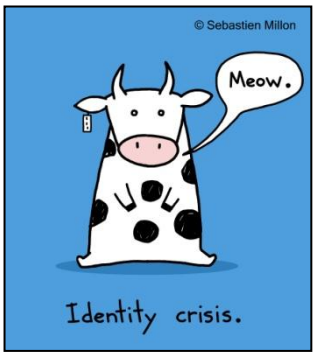
16. $\frac{cotx+cosx}{1+sinx} \equiv cotx$

17. $tanx - cotx \equiv \frac{1-2cos^2x}{sinxcosx}$

18. $\frac{1-cos^2x}{cosx} \cdot cscx \equiv tanx$

19. $2cscx \equiv \frac{sinx}{1+cosx} + \frac{sinx}{1-cosx}$

20. $cosx + sinx \equiv \frac{secx+cscx}{cotx+tanx}$



"Surely you can tell me your secret identity."