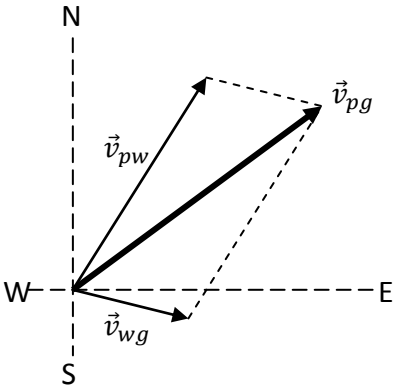
MCV 4U
VELOCITY AS A VECTOR

SPEED is the rate of change of distance with time: $SPEED = \frac{TOTAL\ DISTANCE}{TOTAL\ TIME}$
SPEED is a scalar quantity; it has magnitude but no direction.

VELOCITY is the vector equivalent of speed; it is speed with direction.

RELATIVE VELOCITY is the velocity of an object relative to (or as measured from) a specific **frame of reference**. Vector addition or subtraction is used to calculate relative velocities in different frames of reference.

For example, a **plane's velocity relative to the ground** is the vector sum of the **plane's velocity relative to the wind**, and the **wind's velocity relative to the ground**, as shown below.



The results of the diagram can be summarized as follows:

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$$

This statement is known as the CHAIN RULE.

- $\vec{v}_{pw}$ = the plane's velocity relative to the wind.
  = magnitude is called the **airspeed.**
  = direction is called the **heading**.

- $\vec{v}_{wg}$ = the velocity of the wind relative to the ground.
  = wind's velocity.

- $\vec{v}_{pg}$ = the velocity of the plane relative to the ground.
  = plane's resultant or ground velocity.
  = for a stationary observer from the ground, (s)he views the velocity of the plane.

NOTE: If there is no wind, $\vec{v}_{pw} = \vec{v}_{pg}$.

Relative velocity is important to pilots, canoeists, swimmers, etc., in planning their trips. In addition to the graphical technique, trigonometry and the Pythagorean Theorem can be used to solve relative velocity problems.

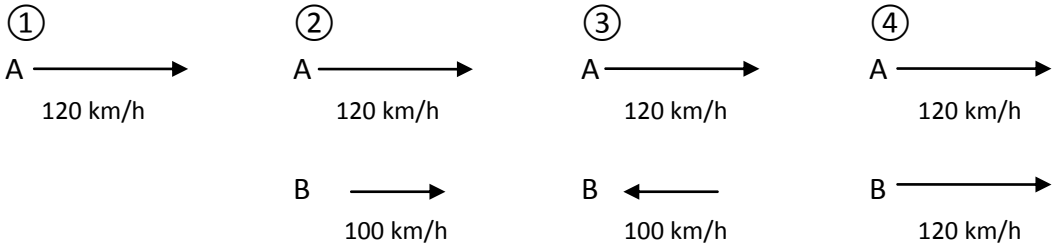
VELOCITY OF ONE OBJECT RELATIVE TO ANOTHER:

For 2 objects A and B moving with velocities $\vec{v}_A$ and $\vec{v}_B$ both relative to the same frame of reference, the chain rule can be used to show that:

- The velocity of B relative to A is given by $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$
- The velocity of A relative to B is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Relative velocity is the difference of two velocities. It is what an observer measures, when (s)he perceives (her)himself to be stationary.

EXAMPLES: Consider 2 vehicles travelling in the same straight lines.



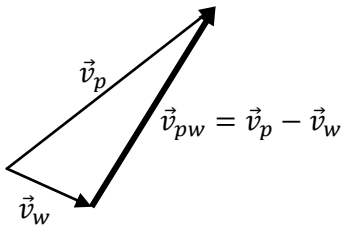
Referring to the CHAIN RULE on the front page $\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$;

isolating $\vec{v}_{pw}$, gives us... $\vec{v}_{pw} = \vec{v}_{pg} - \vec{v}_{wg}$,

but since the ground is not moving we can cross off "g" to simplify.

$\vec{v}_{pw} = \vec{v}_p - \vec{v}_w$

Note how this formula resembles the relative velocities in the formulas above.



Finally, for every **velocity vector** diagram, there is a similar and corresponding **displacement vector** diagram, since time is a common factor to all objects in the velocity diagram.

FRAMES OF REFERENCE & RELATIVE VELOCITY

A frame of reference is a coordinate system relative to which motion can be observed.

For example, two persons sitting together in a train travelling at 50 km/h [E] are in the same frame of reference (the train) and each appears motionless relative to the other. However, a third person standing at the station is in another frame of reference (the earth or ground) and will observe both persons in the train to be moving.

Relative velocity is the velocity of an object relative to a specific frame of reference.

This means while both persons in the train have zero velocity relative to each other, they both have a velocity of 50 km/h [E] relative to the person standing at the station.

Some examples involving relative velocity in two frames of reference:

1. A person swimming in a river that flows relative to the ground.
2. A plane flying in a wind blowing relative to the ground.
3. An astronaut floating around in a spaceship that is orbiting the earth.
4. An astronaut walking on the moon which is orbiting the earth.

EXAMPLES:

① A 70 m wide river flows at 0.80 m/s. A girl swims across it at 1.4 m/s relative to the water.

- A) What is the least time she requires to cross the river?
- B) How far downstream will she be when she lands on the opposite shore?
- C) At what angle to the shore would she have to aim, in order to arrive at a point directly opposite the starting point?
- D) How long would the trip in part (c) take?

② A plane's airspeed is 500 km/h on a heading of N 70° W. A SW wind has a velocity of 40 km/h. Determine the ground speed and direction of the plane.

③ Boat A is travelling at 50 knots on a bearing of 130° while Boat B travels at 75 knots on a bearing of 200°. Determine the relative velocity of Boat A with respect to Boat B.